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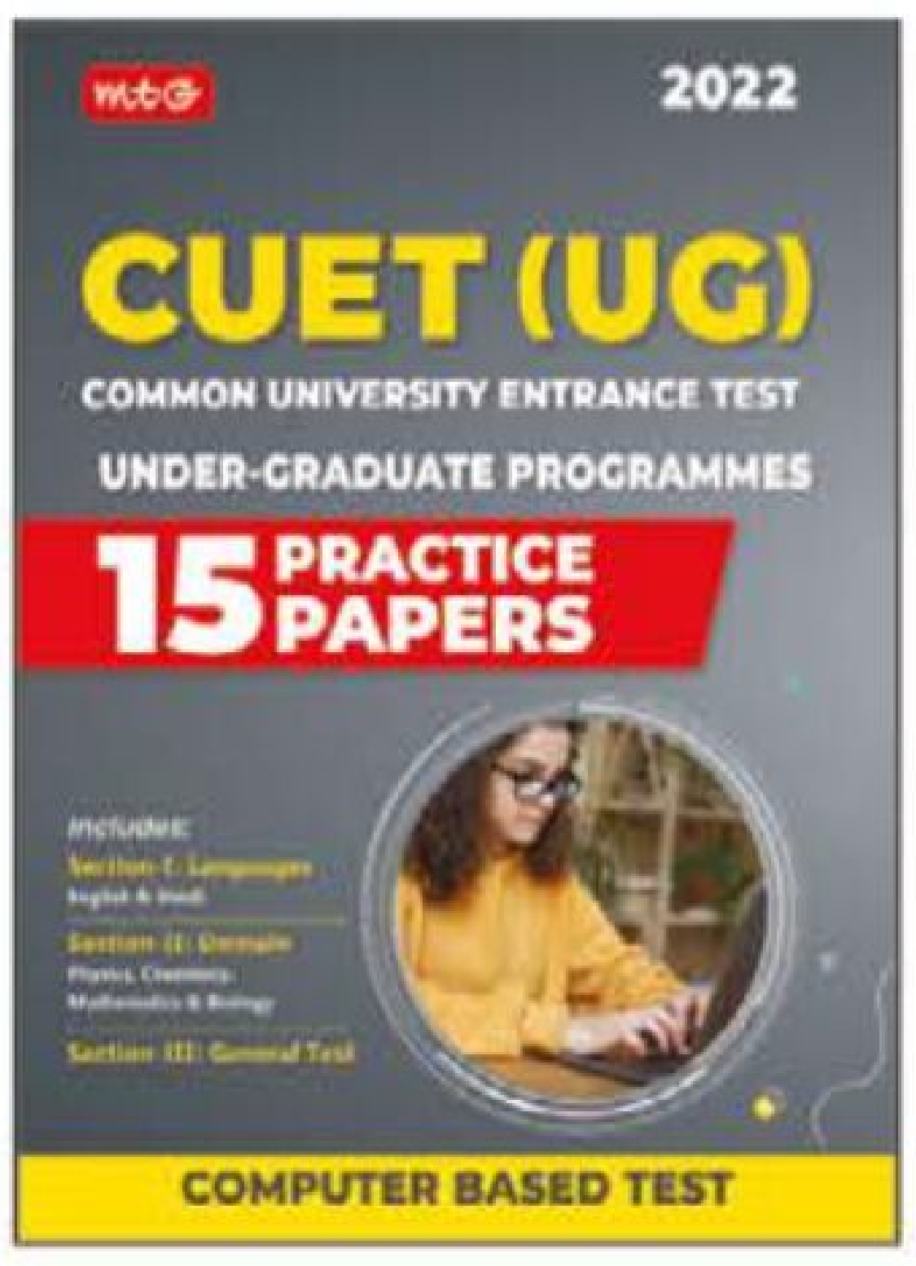
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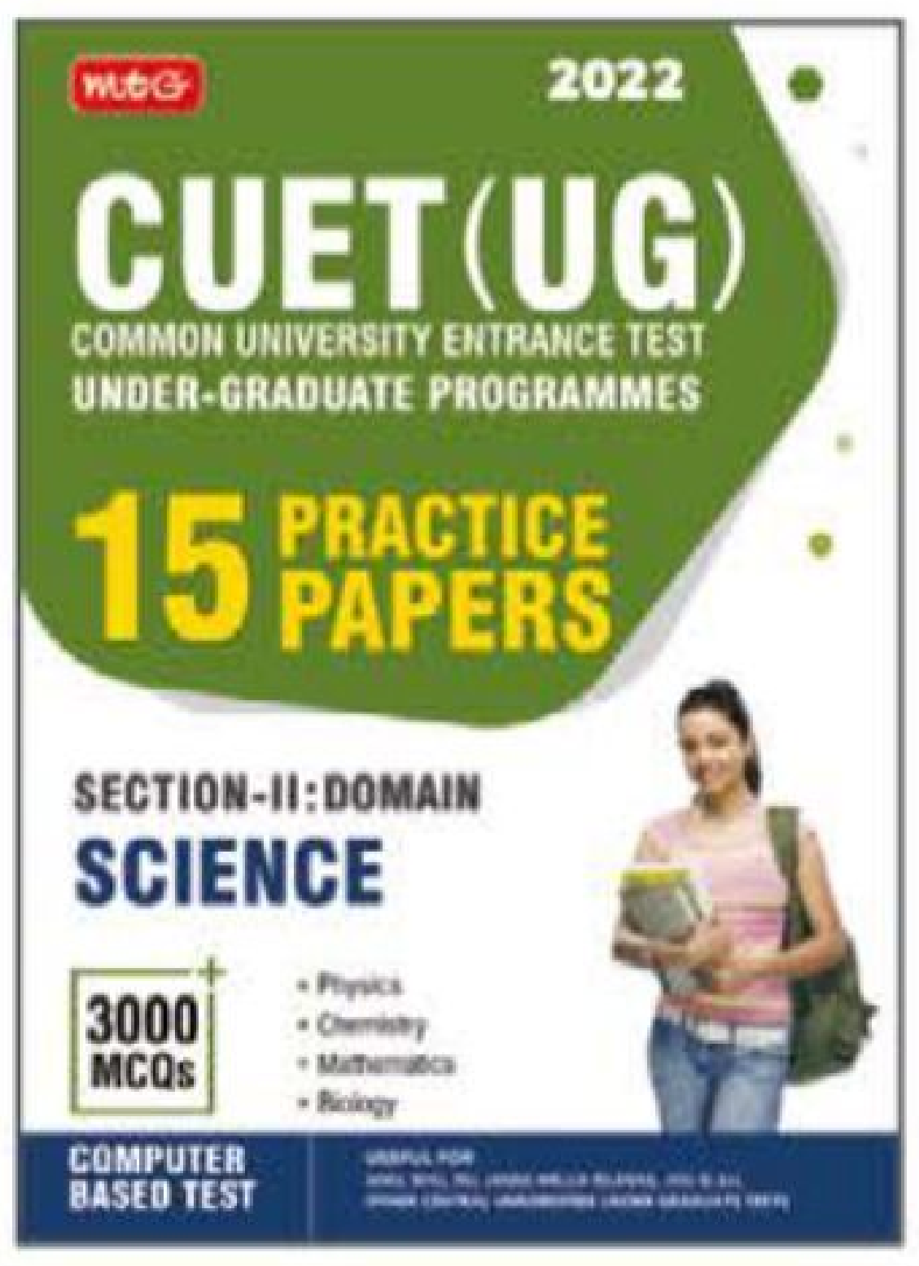
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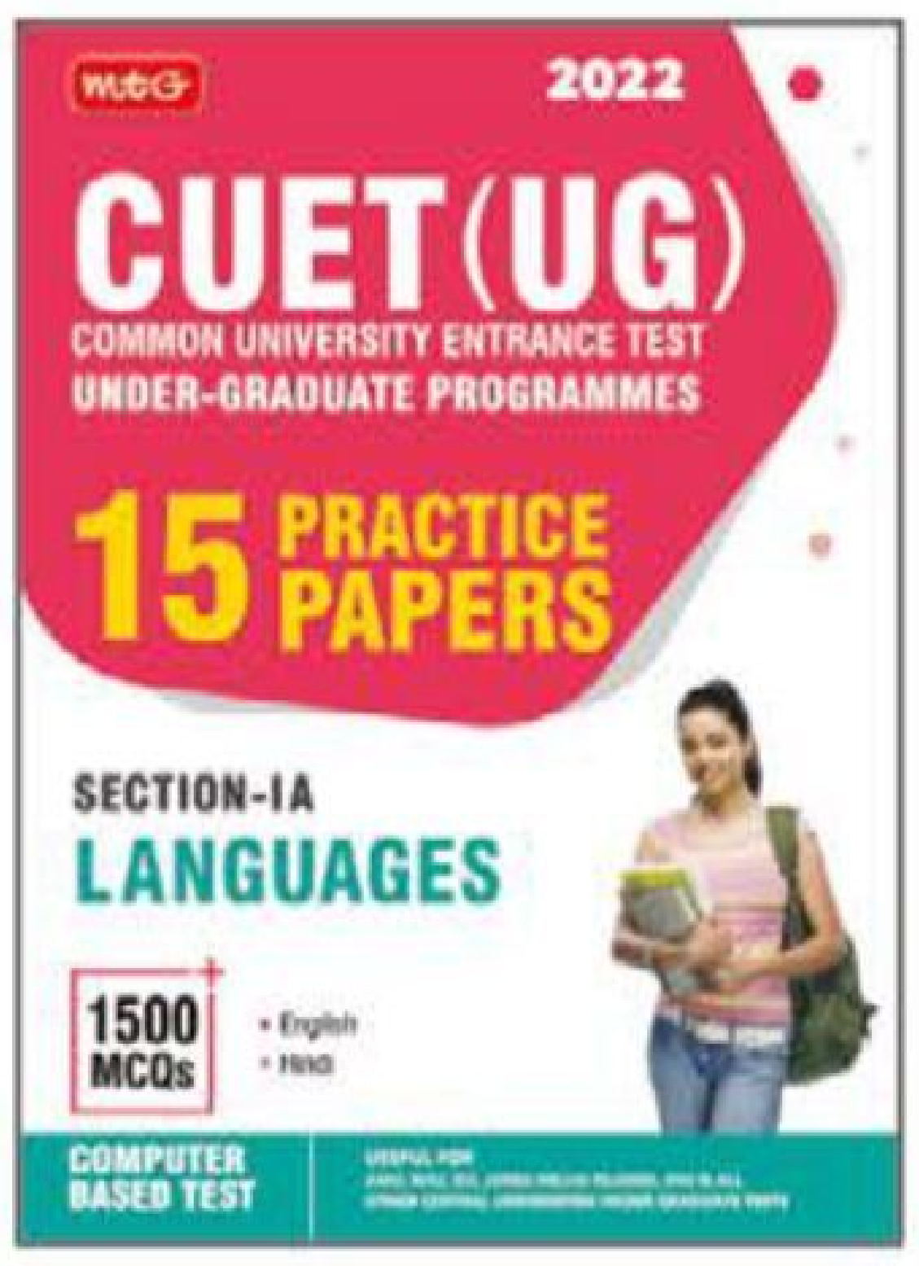
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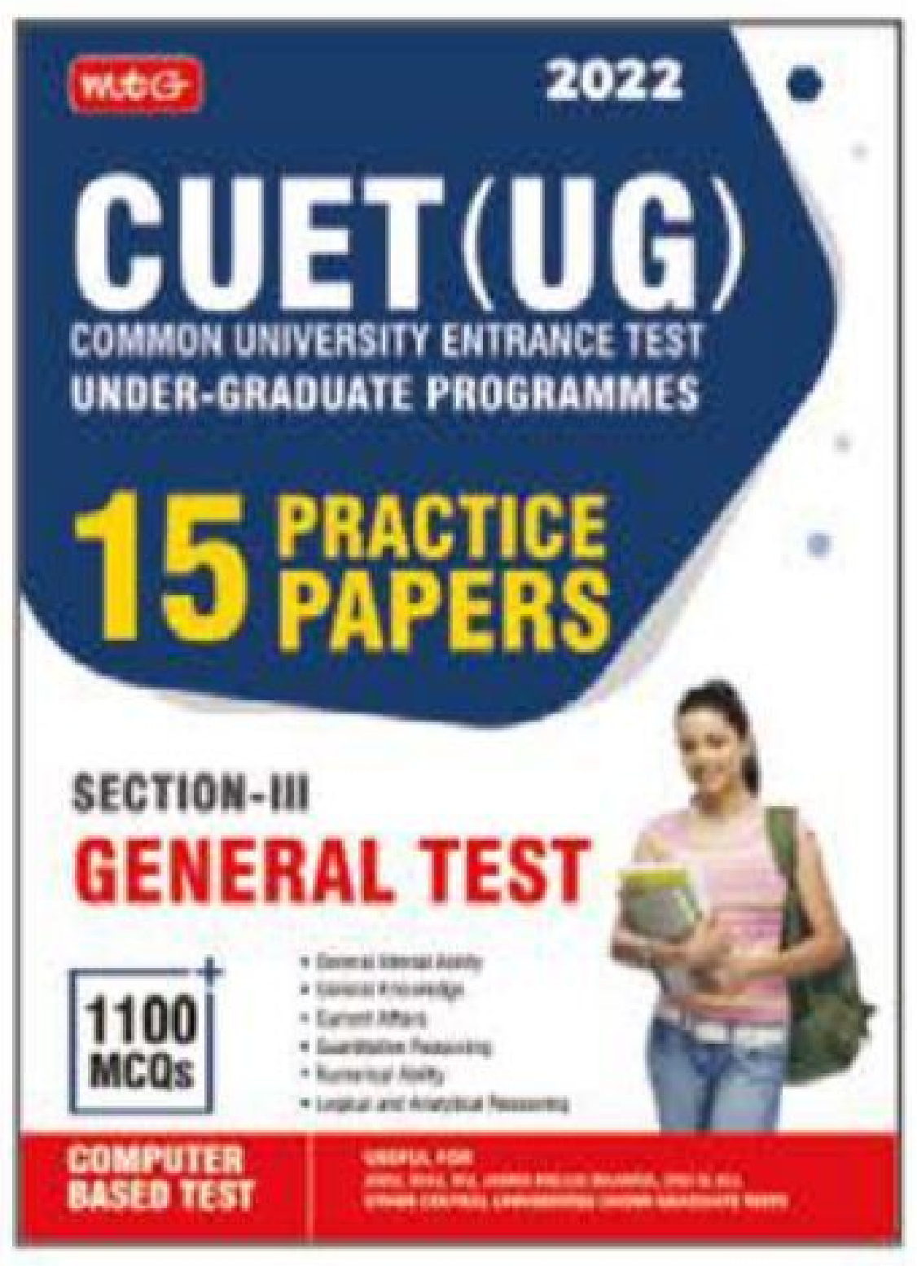
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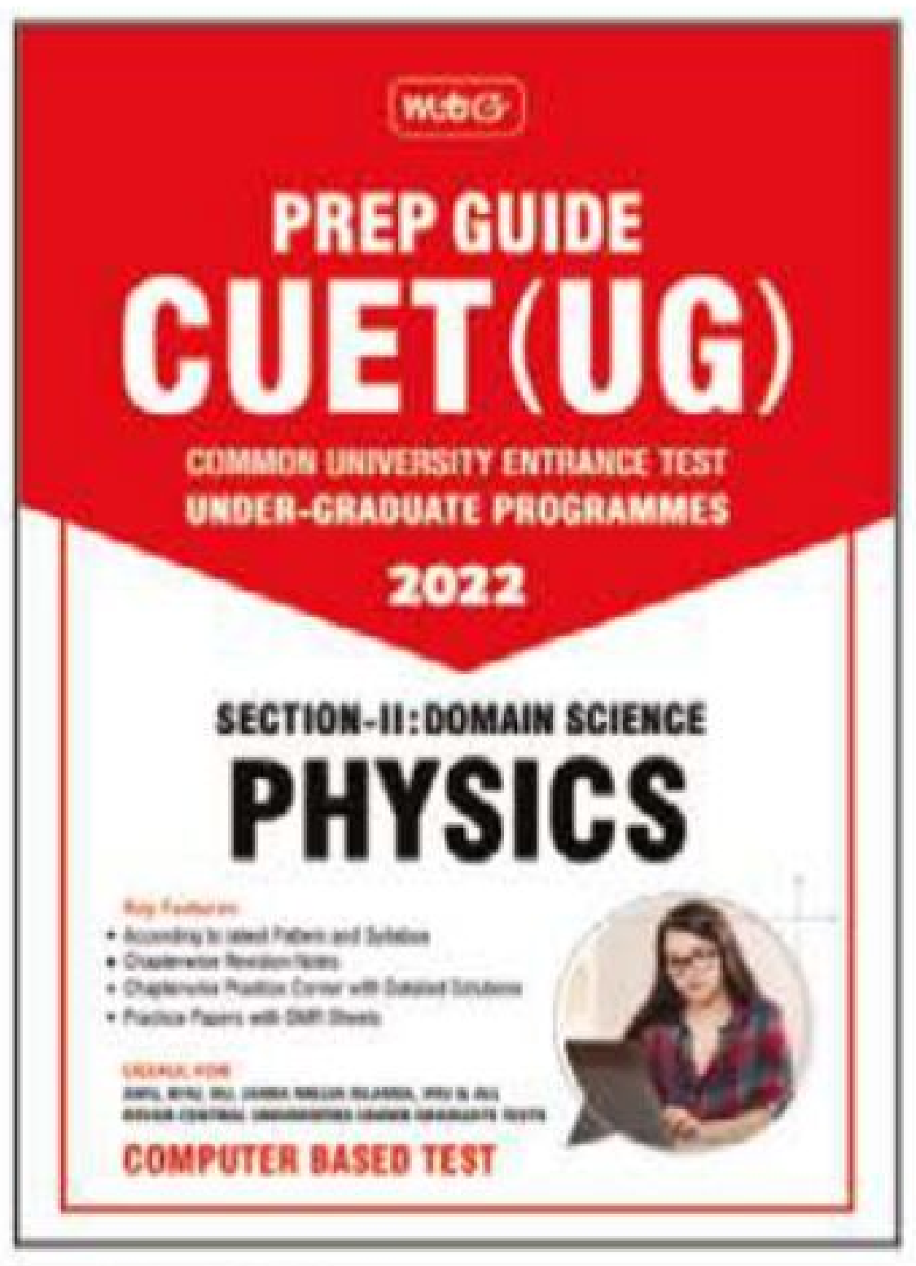
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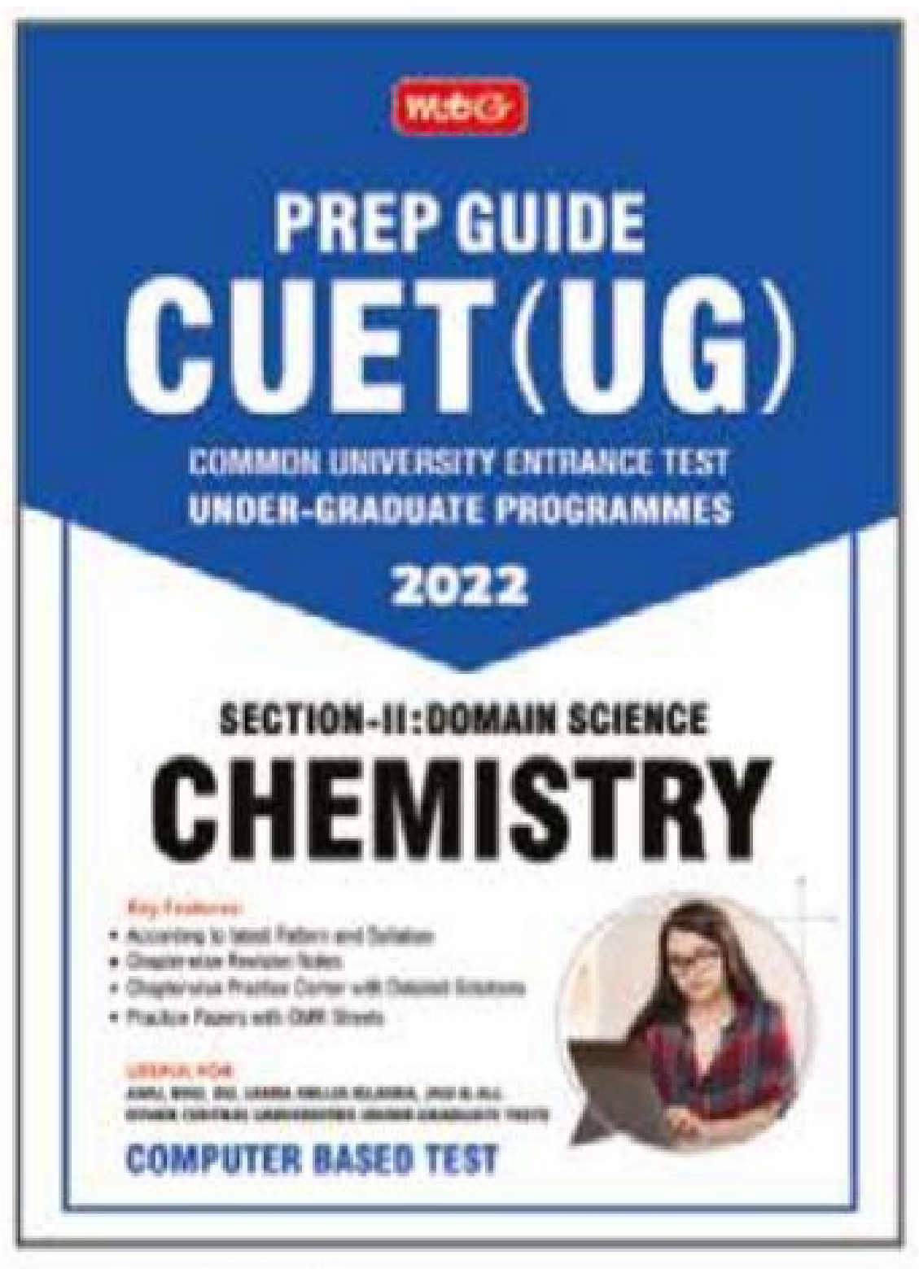
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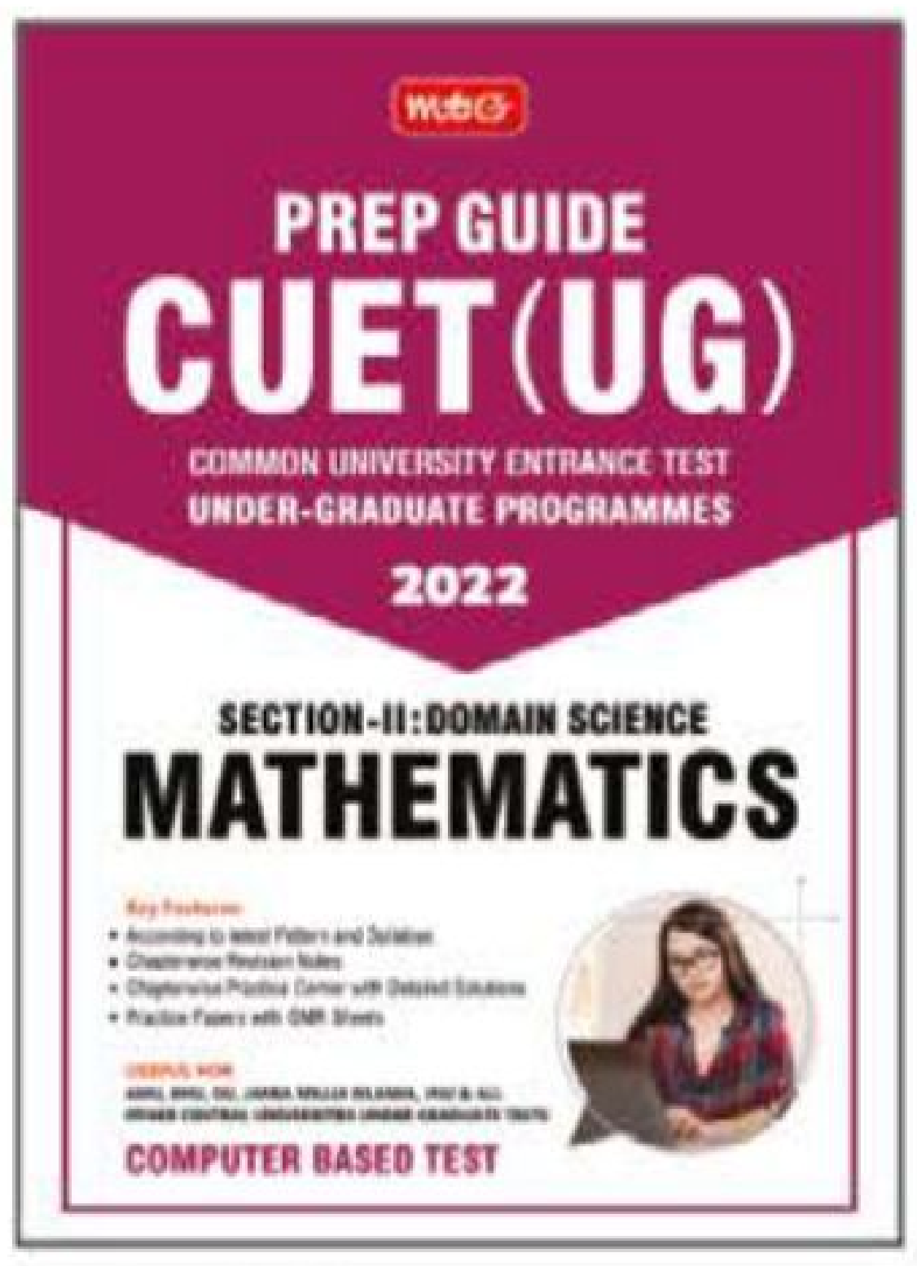
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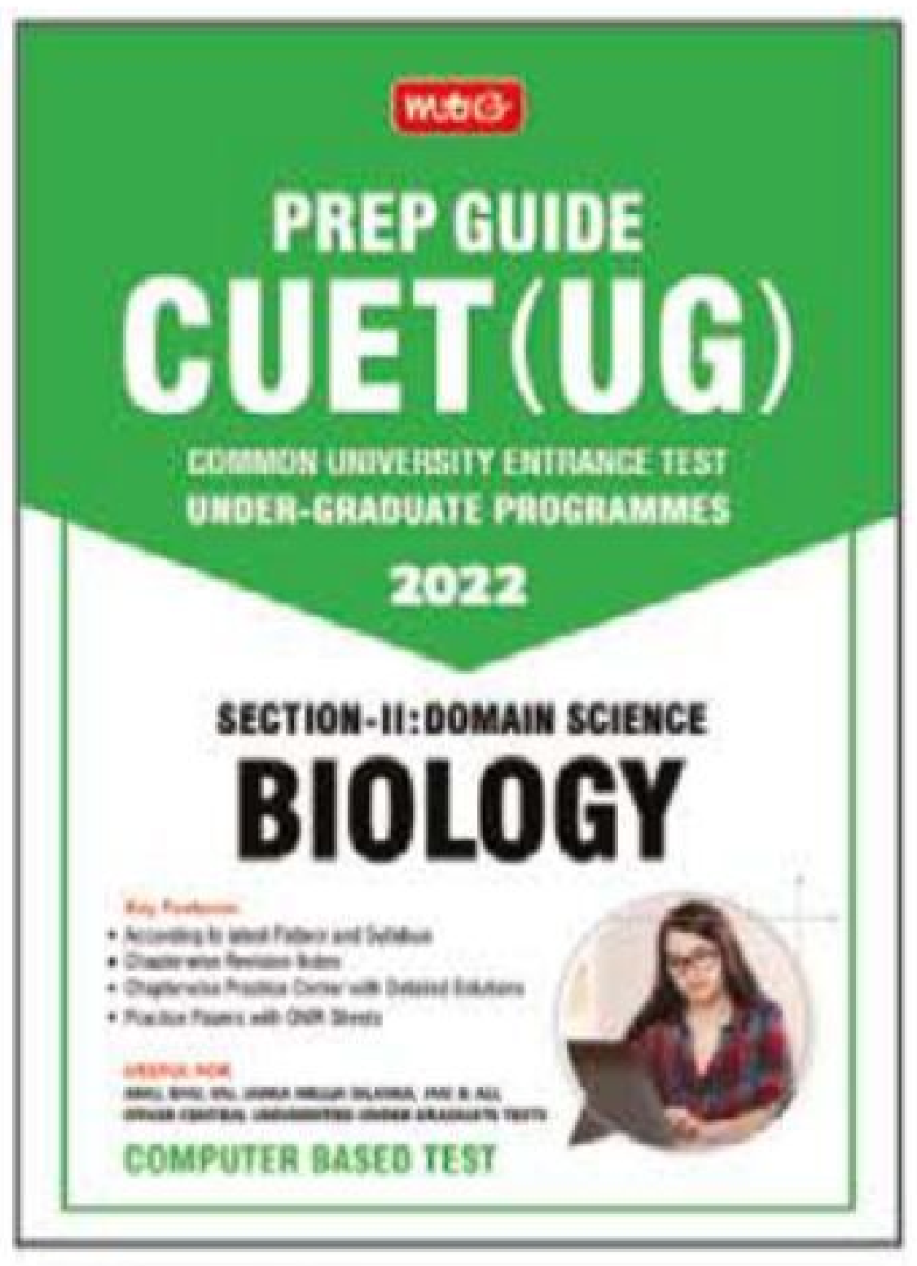
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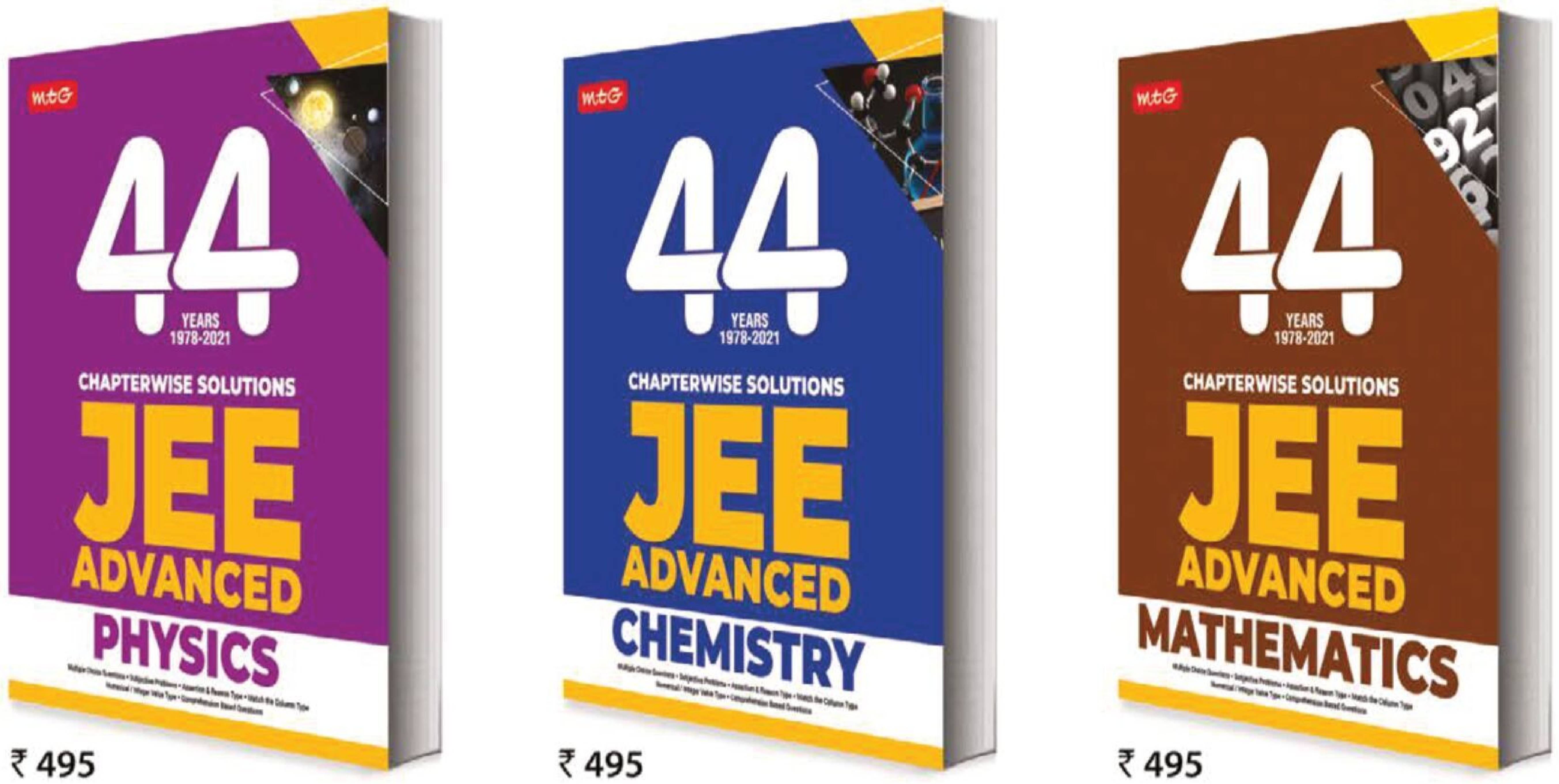

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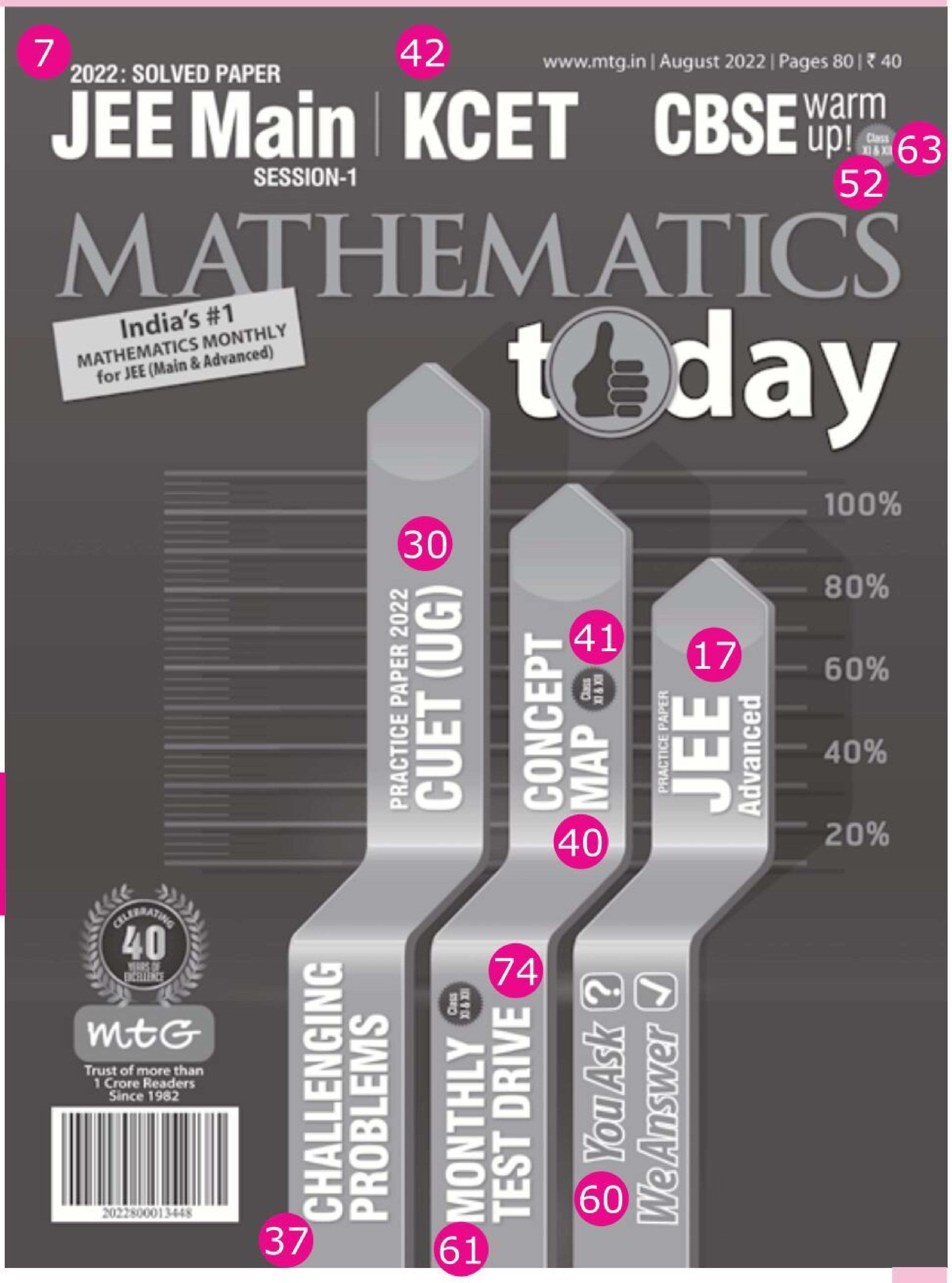
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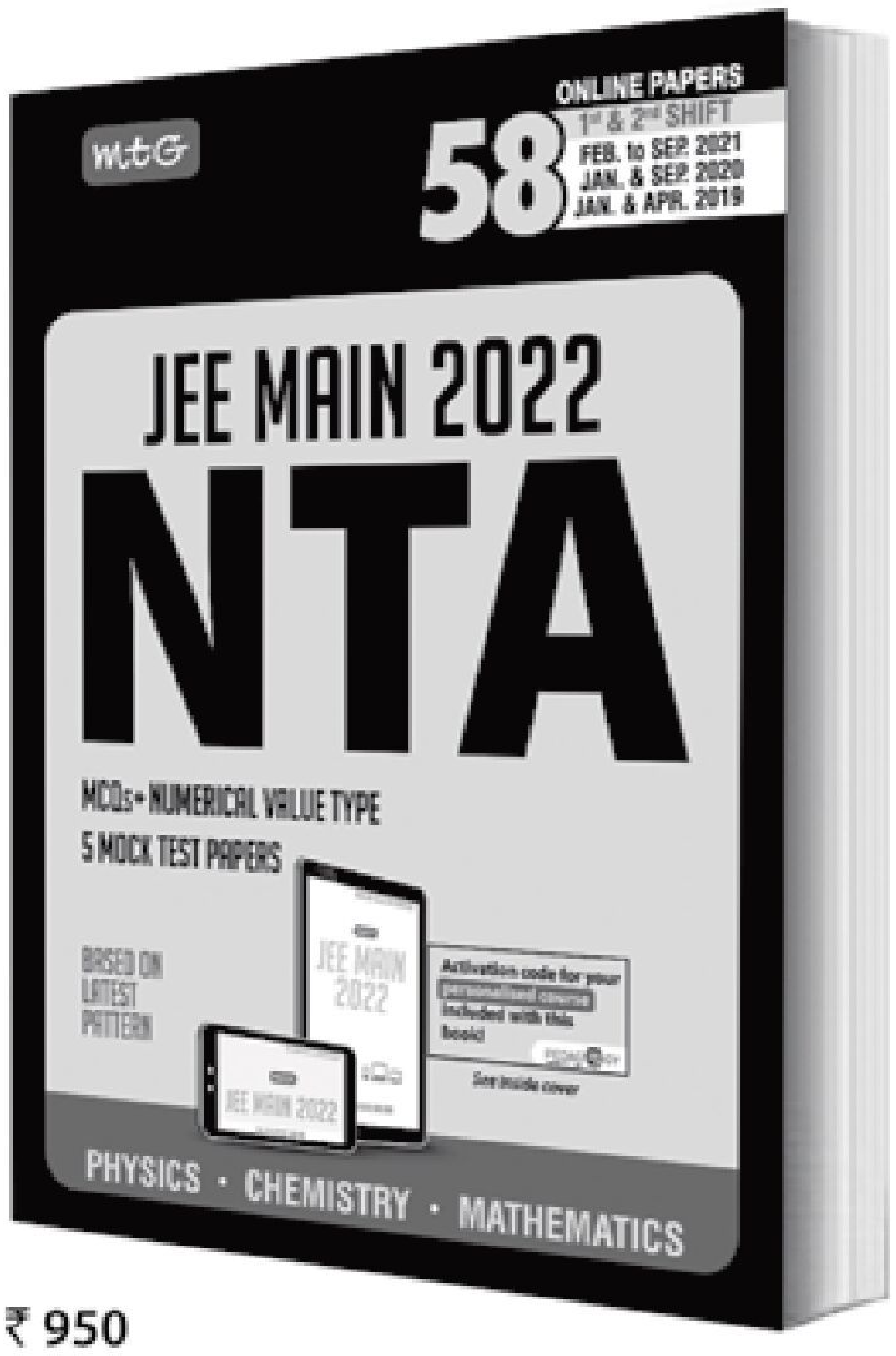
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SOLVED PAPER

Held on
24th June, Morning Shift

JEE MAIN 2022

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. Let $A = \{z \in C : 1 \leq |z - (1+i)| \leq 2\}$ and $B = \{z \in A : |z - (1 - i)| = 1\}$. Then, B

(a) is an empty set
(b) Contains exactly two elements
(c) Contains exactly three elements
(d) is an infinite set

2. The remainder when 3^{2022} is divided by 5 is

(a) 1 (b) 2 (c) 3 (d) 4

3. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds, it becomes 7 units, then its radius after 9 seconds is

(a) 9 (b) 10 (c) 11 (d) 12

4. Bag A contains 2 white, 1 black and 3 red balls and B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random, are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to ______ .

(a) 13 (b) 6 (c) 4 (d) 3

5. Let $x^2 + y^2 + Ax + By + C = 0$ be a circle passing through (0, 6) and touching the parabola $y = x^2$ at (2, 4). Then, $A + C$ is equal to _____ .

(a) 16 (b) 88/5 (c) 72 (d) –8

6. The number of values of α for which the system of equations :

$x + y + z = \alpha$
$\alpha x + 2\alpha y + 3z = -1$
$x + 3\alpha y + 5z = 4$

is inconsistent, is

(a) 0 (b) 1 (c) 2 (d) 3

7. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to

(a) 18 (b) 24
(c) 36 (d) 96

8. The set of all values of k for which $(\tan^{-1} x)^3 + (\cot^{-1} x)^3 = k\pi^3$, $x \in R$, is the interval

(a) $\left[\frac{1}{32}, \frac{7}{8}\right)$ (b) $\left(\frac{1}{24}, \frac{13}{16}\right)$
(c) $\left[\frac{1}{48}, \frac{13}{16}\right]$ (d) $\left[\frac{1}{32}, \frac{9}{8}\right)$

9. Let $S = \{\sqrt{n} : 1 \leq n \leq 50 \text{ and } n \text{ is odd}\}$.

Let $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$.

If $\sum_{a \in S} \det(\operatorname{adj} A) = 100\lambda$, then λ is equal to

(a) 218 (b) 221 (c) 663 (d) 1717

10. For the function $f(x) = 4\log_e(x - 1) - 2x^2 + 4x + 5$, $x > 1$, which one of the following is NOT correct?

(a) f is increasing in (1, 2) and decreasing in $(2, \infty)$
(b) $f(x) = -1$ has exactly two solutions
(c) $f'(e) - f''(2) < 0$
(d) $f(x) = 0$ has a root in the interval $(e, e + 1)$

11. If the tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes though the origin, then (x_1, y_1) does NOT lie on the curve

(a) $x^2 + \frac{y^2}{81} = 2$ (b) $\frac{y^2}{9} - x^2 = 8$
(c) $y = 4x^2 + 5$ (d) $\frac{x}{3} - y^2 = 2$

12. The sum of absolute maximum and absolute minimum values of the function $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval [0, 1] is

(a) $3+\frac{\sin(1)\cos^2(1/2)}{2}$

(b) $3+\frac{1}{2}(1+2\cos(1))\sin(1)$

(c) $5+\frac{1}{2}(\sin(1)+\sin(2))$

(d) $2+\sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

13. If $\{a_i\}_{i=1}^{n}$, where n is an even integer, is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n} a_i = 192, \sum_{i=1}^{n/2} a_{2i} = 120$, and then n is equal to

(a) 48 (b) 96 (c) 92 (d) 104

14. If $x = x(y)$ is the solution of the differential equation $y\frac{dx}{dy} = 2x + y^3(y+1)e^y$, $x(1) = 0$; then $x(e)$ is equal to :

(a) $e^3(e^e - 1)$ (b) $e^e(e^3 - 1)$

(c) $e^2(e^e + 1)$ (d) $e^e(e^2 - 1)$

15. Let $\lambda x - 2y = \mu$ be a tangent to the hyperbola $a^2x^2 - y^2 = b^2$. Then $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is equal to

(a) -2 (b) -4 (c) 2 (d) 4

16. Let $\hat{a}, \hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\vec{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\vec{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|6\,\vec{c}\,|^2$ is equal to :

(a) $6(3-\sqrt{3})$ (b) $3+\sqrt{3}$

(c) $6(3+\sqrt{3})$ (d) $6(\sqrt{3}+1)$

17. If a random variable X follows the Binomial distribution $B(33, p)$ such that $3P(X = 0) = P(X = 1)$ then the value of $\frac{P(x=15)}{p(x=18)} - \frac{P(x=16)}{P(x=17)}$ is equal to

(a) 1320 (b) 1088

(c) $\frac{120}{1331}$ (d) $\frac{1088}{1089}$

18. The domain of the function

$$f(x) = \frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\log_e(x^2-3x+2)} \text{ is}$$

(a) $(-\infty, 1) \cup (2, \infty)$

(b) $(2, \infty)$

(c) $[-1/2, 1) \cup (2, \infty)$

(d) $[-1/2, 1) \cup (2, \infty) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

19. Let $S = \left\{\theta \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}$

If $T = \sum_{\theta \in S} \cos 2\theta$, then $T + n(S)$ is equal to

(a) $7+\sqrt{3}$ (b) 9

(c) $8+\sqrt{3}$ (d) 10

20. The number of choices for $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow,\}$, such that $(p\,\Delta\,q) \Rightarrow ((p\,\Delta \sim q) \vee ((\sim p)\,\Delta\,q))$ is a tautology, is

(a) 1 (b) 2

(c) 3 (d) 4

SECTION-B (NUMERICAL TYPE QUESTIONS

Attempt any 5 questions out of 10.

21. The number of one-one function $f : \{a, b, c, d\} \rightarrow \{0, 1, 2, ..., 10\}$ such that $2f(a) - f(b) + 3f(c) + f(d) = 0$ is _____ .

22. In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct. There are 3 marks for each correct answer, -2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is ______ .

23. Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right), a > 0$, be a fixed point in the xy-plane. The image of A in y-axis be B and the image of B in x-axis be C. If $D(3\cos\theta, a\sin\theta)$ is a point in the fourth quadrant such that the maximum area of ΔACD is 2 square units, then a is equal to ______ .

24. Let a line having direction rations 1, -4, 2 intersect the lines $\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ and $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ at the points A and B. Then $(AB)^2$ is equal to

25. The number of points where the function

$$f(x) = \begin{cases} (2x^2-3x-7) & \text{if } x \le -1 \\ [4x^2-1] & \text{if } -1 < x < 1 \\ |x+1| + |x-2| & \text{if } x \ge 1, \end{cases}$$

$[t]$ denotes the greatest integer $\leq t$, is discontinuous is ______ .

26. Let $f(\theta)=\sin\theta+\int\limits_{-\pi/2}^{\pi/2}(\sin\theta+t\cos\theta)f(t)dt$. Then the value of $\left|\int_0^{\pi/2}f(\theta)d\theta\right|$ is ______ .

27. Let $\underset{0\leq x\leq 2}{\text{Max}}\left\{\frac{9-x^2}{5-x}\right\}=\alpha$ and $\underset{0\leq x\leq 2}{\text{Min}}\left\{\frac{9-x^2}{5-x}\right\}=\beta$.

If $\int\limits_{\beta-\frac{8}{3}}^{2\alpha-1}\text{Max}\left\{\frac{9-x^2}{5-x},x\right\}dx=\alpha_1+\alpha_2\log_e\left(\frac{8}{15}\right)$ then $\alpha_1+\alpha_2$ is equal to ______ .

28. If two tangents drawn from a point (α, β) lying on the ellipse $25x^2+4y^2=1$ to the parabola $y^2=4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha+5)^2+(16\beta^2+50)^2$ equals ______ .

29. Let S be the region bounded by the curves $y=x^3$ and $y^2=x$. The curve $y=2|x|$ divides S into two regions of area R_1 and R_2.

If max $\{R_1, R_2\}=R_2$, then $\frac{R_2}{R_1}$ is equal to ______ .

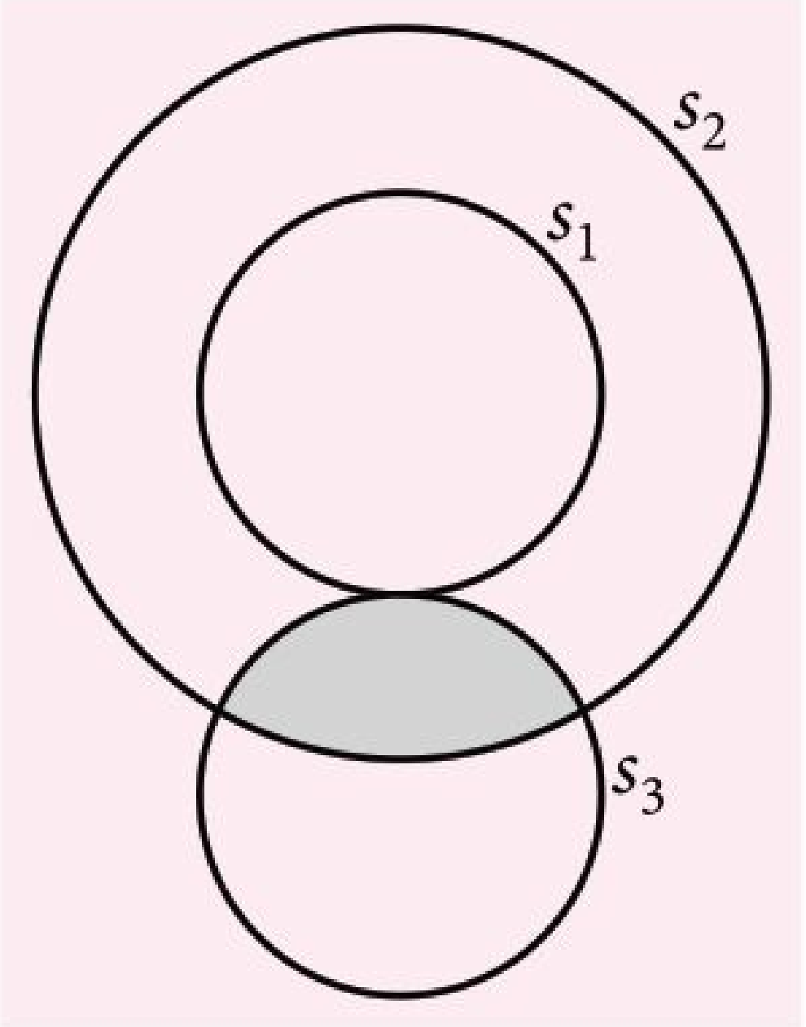
30. If the shortest distance between the lines $\vec{r}=(-\hat{i}+3\hat{k})+\lambda(\hat{i}-a\hat{j})$ and $\vec{r}=(-\hat{j}+2\hat{k})+\mu(\hat{i}-\hat{j}+\hat{k})$ is $\sqrt{\frac{2}{3}}$, then the integral value of a is equal to _____ .

SOLUTIONS

1. (d): Here,

$1\leq|z-(1+i)|\leq 2$

which represents two concentric circles s_1 and s_2 having centre $(1, 1)$ and radii 1 and 2 respectively. Also, $|z-(1-i)|=1$ also represents circle s_3 with centre $(1, -1)$ and radius 1.



So, from figure, B has all the elements which lie in the shaded region.

So, B is an infinite set.

2. (d): We have, $3^{2022}=3^{4\times 505+2}$

$=(81)^{505}\times 3^2$

$=(81)^{505}\times 9$

So, remainder $=4$

3. (a): Let A be the surface area of balloon.

According to question, we have

$$\frac{dA}{dt}=k$$

Integrating both sides, we get

$A=kt+C$

$\Rightarrow$ $4\pi r^2=kt+C$ ...(i), where r is the radius of balloon.

Now, at $t=0$, $r=3$ units

$\therefore$ $36\pi=C$ ...(ii) (Using (i))

Also, at $t=5$ sec, $r=7$ units

$\therefore$ $196\pi=5k+36\pi$ (Using (i) & (ii))

$\Rightarrow$ $k=32\pi$

So, from (i), we have

$4\pi r^2=32\pi\, t+36\pi$

$\Rightarrow$ $r^2=8t+9$

$\therefore$ At $t=9$ secs, we have

$r=\sqrt{8\times 9+9}=9$ units

4. (c): Let E_1 be the event of drawing 1 red and 1 black ball and E be the event that ball is drawn from bag A.

$$\therefore P(E)=\frac{1}{2}, P(\bar{E})=\frac{1}{2},\quad P(E_1|E)=\frac{{}^1C_1\times{}^3C_1}{{}^6C_2}=\frac{1}{5}$$

$$P(E_1|\bar{E})=\frac{{}^3C_1\times{}^2C_1}{{}^{5+n}C_2}=\frac{12}{(5+n)(4+n)}$$

Now, by Baye's Theorem

$$P(E|E_1)=\frac{P(E)\cdot P(E_1|E)}{P(E)\cdot P(E_1|E)+P(\bar{E})\cdot P(E_1|\bar{E})}$$

$$\Rightarrow\quad \frac{6}{11}=\frac{\frac{1}{2}\cdot\frac{1}{5}}{\frac{1}{2}\cdot\frac{1}{5}+\frac{1}{2}\cdot\frac{12}{(5+n)(4+n)}}$$

$$\Rightarrow\quad \frac{6}{11}=\frac{(5+n)(4+n)}{(5+n)(4+n)+60}$$

$\Rightarrow$ $5(5+n)(4+n)=360=5\cdot(5+4)(4+4)$

$\Rightarrow$ $n=4$

5. (a): The equation of circle is,

$x^2+y^2+Ax+By+C=0$ ...(i)

Differentiate (i) w.r.t x, we get

$$2x+2y\frac{dy}{dx}+A+B\frac{dy}{dx}=0\quad\Rightarrow\quad\frac{dy}{dx}=-\left(\frac{2x+A}{2y+B}\right)$$

So, slope of tangent to circle at $(2, 4)$ is,

$$m_1=-\left(\frac{4+A}{8+B}\right)$$

Also, equation of parabola is,

$$y=x^2\;\Rightarrow\;\frac{dy}{dx}=2x$$

So, slope of tangent to parabola at (2, 4) is,

$m_2 = 4$

Now, $m_1 = m_2$ (∵ Circle touches the parabola at (2, 4))

$$\therefore \quad 4 = \frac{-4-A}{8+B}$$

$\Rightarrow$ $32 + 4B = -4 - A \Rightarrow A + 4B = -36$ ...(ii)

Also, circle passes through (2, 4)

$\therefore$ $4 + 16 + 2A + 4B + C = 0$

$\Rightarrow$ $2A + 4B + C = -20$

$\Rightarrow$ $A + (-36) + C = -20$ (From (ii))

$\Rightarrow$ $A + C = 16$

6. (b): For given system of equations is inconsistent, if $D = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix} = 0 \quad \Rightarrow \quad \alpha^2 - 2\alpha + 1 = 0$$

$\Rightarrow$ $(\alpha - 1)^2 = 0 \Rightarrow \alpha = 1$

So, there exists only one value of α for which the given system of equations is inconsistent.

7. (b): ∵ α, β are the roots of $3x^2 + \lambda x - 1 = 0$

$$\Rightarrow \alpha + \beta = -\frac{\lambda}{3} \qquad \text{...(i)}$$

$$\Rightarrow \alpha\beta = -\frac{1}{3} \qquad \text{...(ii)}$$

Now, it is given that, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$

$$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = 15$$

$$\Rightarrow \frac{\lambda^2}{9} + \frac{2}{3} = 15\left(\frac{1}{9}\right) \qquad \text{[Using (i) \& (ii)]}$$

$\Rightarrow$ $\lambda = \pm 3$ ...(iii)

Now, $6(\alpha^3 + \beta^3)^2 = 6[(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)]^2$

$$= 6\left[\frac{-\lambda^3}{27} + 3 \cdot \frac{1}{3} \cdot \left(\frac{-\lambda}{3}\right)\right]^2$$

$= 6[\mp 1 \mp 1]^2 = 6 \times 4 = 24$

8. (a): We have, $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 = k\pi^3$ ...(i)

Let $y = (\tan^{-1}x)^3 + (\cot^{-1}x)^3$

$= (\tan^{-1}x + \cot^{-1}x)\,[(\tan^{-1}x)^2 + (\cot^{-1}x)^2 - \tan^{-1}x \cdot \cot^{-1}x]$

$$= \frac{\pi}{2}\left[t^2 + \left(\frac{\pi}{2} - t\right)^2 - t\left(\frac{\pi}{2} - t\right)\right] \qquad \text{[where } t = \tan^{-1}x]$$

$$= \frac{\pi}{2}\left[3t^2 - \frac{3}{2}\pi t + \frac{\pi^2}{4}\right]$$

Now, for maximum/minimum, put $y'(t) = 0$

$$\Rightarrow 6t - \frac{3\pi}{2} = 0 \Rightarrow t = \frac{\pi}{4}$$

So, $y_{\min}$ at $t = \frac{\pi}{4}$ and $y_{\max}$ at $t = -\frac{\pi}{2}$

$$\therefore \quad y_{\min} = \frac{\pi}{2}\left[\frac{3\pi^2}{16} - \frac{3\pi^2}{8} + \frac{\pi^2}{4}\right] = \frac{\pi^3}{32}$$

$$\text{and } y_{\max} = \frac{\pi}{2}\left[\frac{3\pi^2}{4} + \frac{3\pi^2}{4} + \frac{\pi^2}{4}\right] = \frac{7\pi^3}{8}$$

From (i), we have,

$$\frac{\pi^3}{32} \le k\pi^3 < \frac{7}{8}\pi^3$$

$$\Rightarrow \frac{1}{32} \le k < \frac{7}{8} \Rightarrow k \in \left[\frac{1}{32}, \frac{7}{8}\right)$$

9. (b): We have, $S = \{\sqrt{n} : 1 \le n \le 50, n \text{ is odd}\}$

$\Rightarrow$ $n = \{1, 3, 5, ..., 49\}$

Now, $|\text{adj } A| = |A|^{n-1} = |A|^2$

Here, $|A| = 1\,(1 - 0) - 0 + a\,(0 + a) = 1 + a^2$

$\therefore$ $|\text{adj } A| = (1 + a^2)^2$

$$\Rightarrow \sum_{a \in S} |adjA| = \sum (1+n)^2 \qquad [\because \alpha^2 = n \in S]$$

$\Rightarrow$ $100\lambda = 2^2 + 4^2 + ... + 50^2$

$$= 2^2(1 + 2^2 + ... + 25^2) = \frac{4 \cdot 25 \cdot 26 \cdot 51}{6}$$

$\Rightarrow$ $100\lambda = 22100$

$\Rightarrow$ $\lambda = 221$

10. (c): We have,

$f(x) = 4\log_e(x - 1) - 2x^2 + 4x + 5$

$$f'(x) = \frac{4}{x-1} - 4x + 4 \qquad \text{...(i)}$$

$$\text{and } f''(x) = \frac{-4}{(x-1)^2} - 4 \qquad \text{...(ii)}$$

Now, $f'(x) > 0$ for increasing and $f'(x) < 0$ for decreasing

$$\Rightarrow \frac{2x - x^2}{x-1} > 0, \frac{2x - x^2}{x-1} < 0 \qquad \text{[Using (i)]}$$

$\Rightarrow$ $x \in (-\infty, 0) \cup (1, 2), x \in (0, 1) \cup (2, \infty)$

So, option (a) is correct.

If $f(x) = -1 \Rightarrow 4\log_e(x - 1) = 2x^2 - 4x - 6$

$\Rightarrow$ $4\log_e(x - 1) = 2\,(x - 3)\,(x + 1)$

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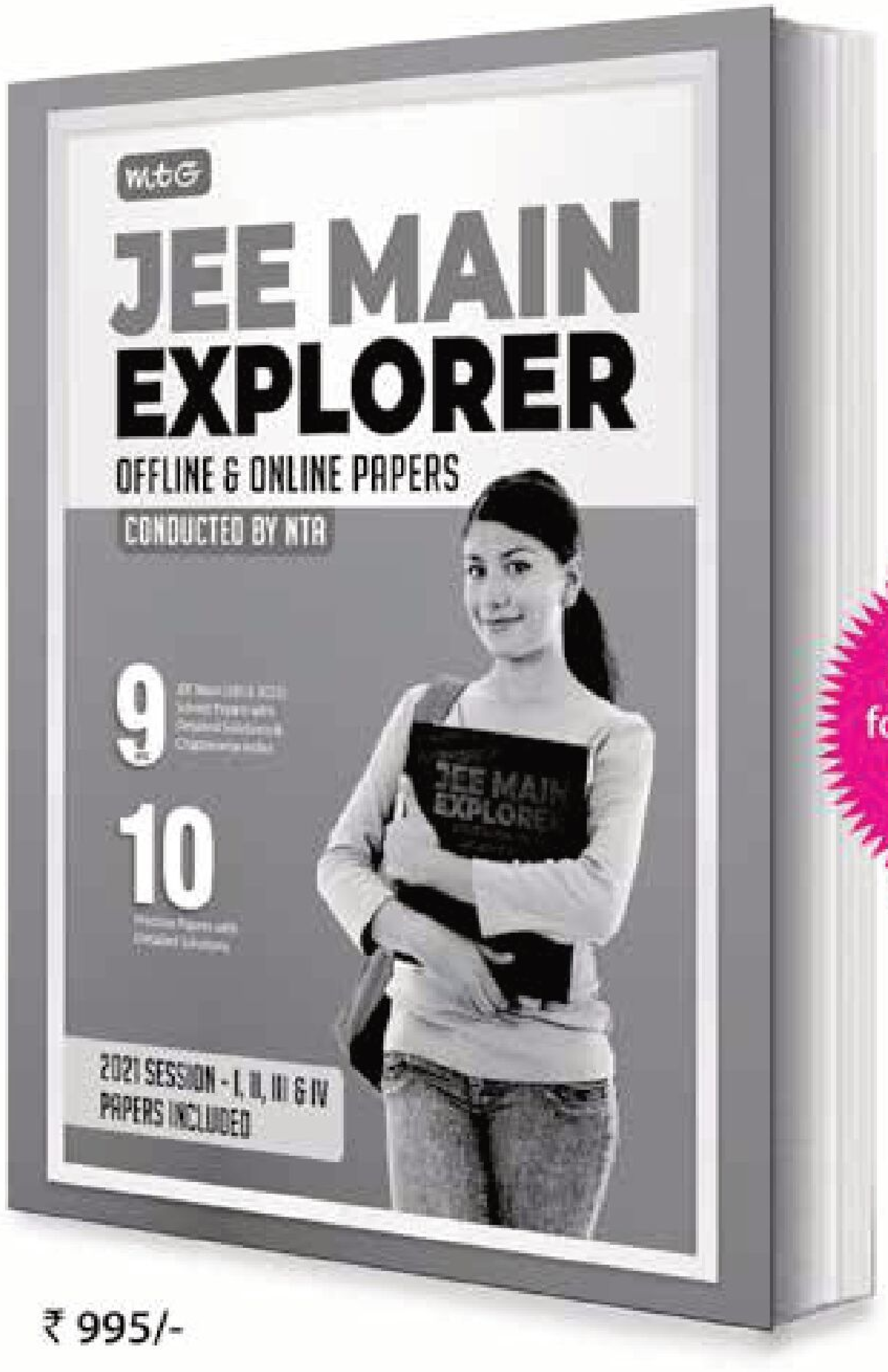
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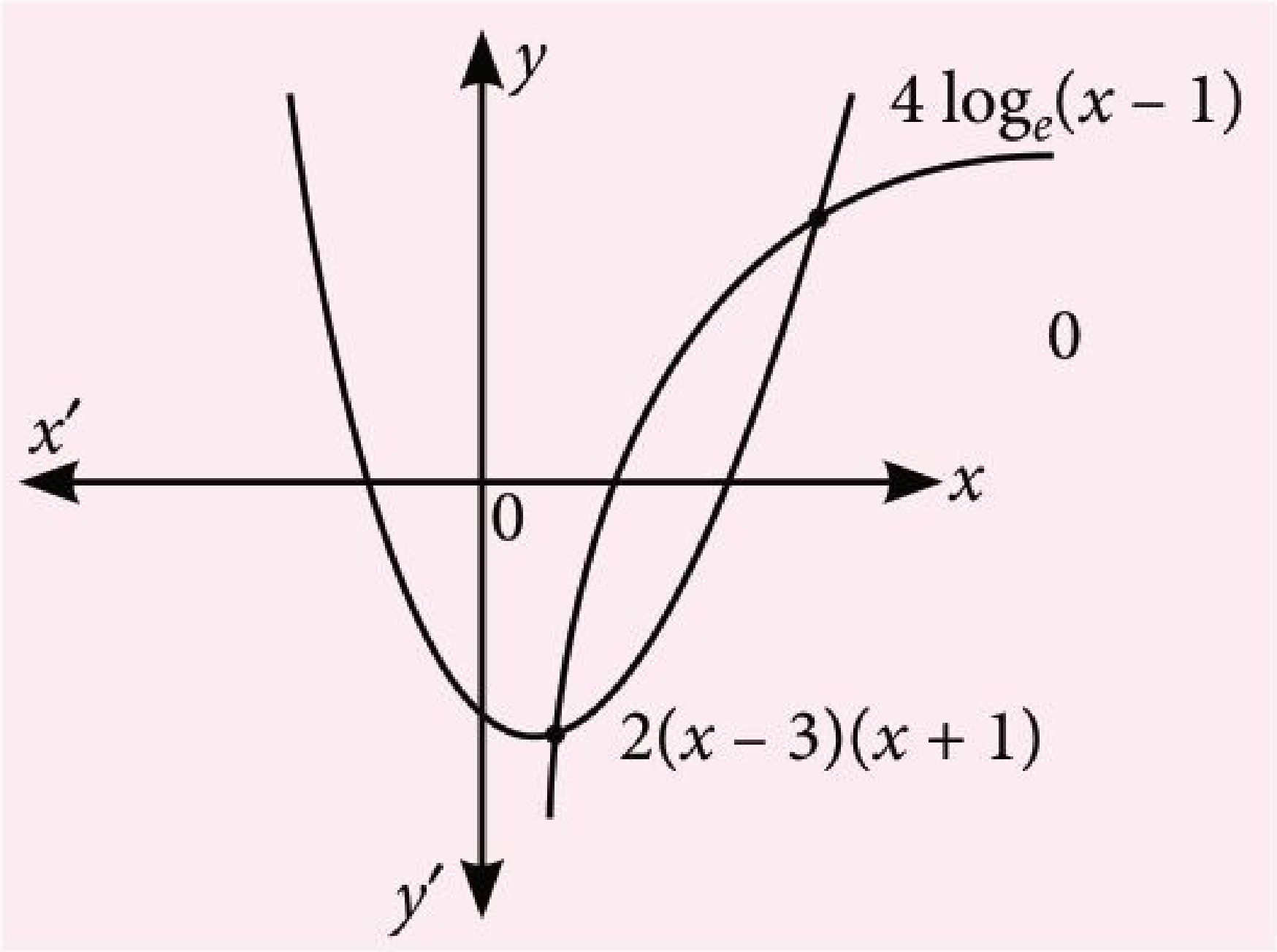


From the graph, it is clear that the above equation has exactly 2 solutions.

Now, $f'(e) - f''(2)$

$$= 4\left(\frac{2e-e^2}{e-1}\right)+4+4>0$$

so, (c) is false.

Also, (d) is correct.

11. (d): The given equation of curve is,

$y = x^3 + 3x^2 + 5$ ...(i)

$y' = 3x^2 + 6x$

Now, equation of tangent at (x_1, y_1) to the curve is, $y - y_1 = (3x_1^2 + 6x_1)(x - x_1)$

$\because$ Tangent passes through origin

$\therefore$ $y_1 = 3x_1^3 + 6x_1^2$ ...(ii)

Also, we have, $y_1 = x_1^3 + 3x_1^2 + 5$ ...(iii)

solving (ii) and (iii), we get

$2y_1 = 3x_1^2 + 15$

$\therefore$ Equation of curve is,

$$y=\frac{3}{2}x^2+\frac{15}{2}$$

This curve does not intersect $\frac{x}{3} - y^2 = 2$.

12. (b): We have,

$f(x) = |2x^2 + 3x - 2| + \sin x \cos x, x \in [0, 1]$

$\Rightarrow f(x) = |(2x - 1)(x + 2)| + \sin x \cos x$

$$\Rightarrow f(x)=\begin{cases}-(2x-1)(x+2)+\sin x\cos x, & x\in\left[0,\frac{1}{2}\right]\\(2x-1)(x+2)+\sin x\cos x, & x\in\left[\frac{1}{2},1\right]\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}-2x^2-3x+2+\frac{1}{2}\sin 2x, & x\in\left[0,\frac{1}{2}\right]\\2x^2+3x-2+\frac{1}{2}\sin 2x, & x\in\left[\frac{1}{2},1\right]\end{cases}$$

$$\Rightarrow f'(x)=\begin{cases}-4x-3+\cos 2x, & x\in\left[0,\frac{1}{2}\right)\\4x+3+\cos 2x, & x\in\left(\frac{1}{2},1\right]\end{cases}$$

$$\Rightarrow f'(x)<0 \text{ for } x\in\left[0,\frac{1}{2}\right)$$

$$\therefore\ f_{\min}=f\left(\frac{1}{2}\right)=-2\left(\frac{1}{4}\right)-3\left(\frac{1}{2}\right)+2+\frac{1}{2}\sin 1=\frac{1}{2}\sin 1$$

Similarly, $f'(x) > 0$ for $x\in\left(\frac{1}{2},1\right]$

$$\therefore\ f_{\max}=f(1)=3+\frac{1}{2}\sin 2=3+\sin 1\cos 1$$

$$\therefore\ \text{Required sum }=f\left(\frac{1}{2}\right)+f(1)$$

$$=\frac{1}{2}\sin 1+3+\sin 1\cos 1=3+\frac{1}{2}\sin 1(1+2\cos 1)$$

13. (b): Here, $d = 1$, $a_1, a_2, a_3 \ldots a_n \in$ A.P.

$$\sum_{i=1}^{n}a_i=192$$

$$\Rightarrow \frac{n}{2}\left[2a_1+(n-1)1\right]=192$$

$$\Rightarrow na_1+\frac{n(n-1)}{2}=192$$

$$\Rightarrow na_1=192-\frac{n(n-1)}{2} \qquad \text{...(i)}$$

Also, $\sum_{i=1}^{n/2}a_{2i}=120$

$\Rightarrow a_2 + a_4 + \ldots + a_n = 120$

$\Rightarrow a_1 + 1 + a_1 + 3 + \ldots. + a_1 + (n - 1) 1 = 120$

$\Rightarrow \frac{n}{2}(a_1) + [1 + 3 + \ldots. + (n - 1)] = 120$

$$\Rightarrow \frac{n}{2}(a_1)+\left(\frac{n}{2}\right)^2=120$$

[Sum of first n odd natural numbers = n^2]

$$\Rightarrow \frac{1}{2}\left(192-\frac{n(n-1)}{2}\right)+\frac{n^2}{4}=120 \qquad \text{[Using (i)]}$$

$$\Rightarrow 96+\frac{n}{4}=120 \Rightarrow n=96$$

14. (a): The given differential equation can be rewritten as, $\frac{dx}{dy}-\frac{2x}{y}=y^2(y+1)e^y$, which is a linear D.E.

$$\therefore \quad \text{I.F.} = e^{-\int \frac{2}{y} dy} = \frac{1}{y^2}$$

So, solution of given D.E. is,

$$\frac{x}{y^2} = \int (y+1)e^y\, dy + C$$

$$\Rightarrow \frac{x}{y^2} = \int y e^y\, dy + \int e^y\, dy + C \Rightarrow \frac{x}{y^2} = y \cdot e^y + C$$

$\Rightarrow x = y^2(y \cdot e^y + C)$ ...(i)

Now, $x(1) = 0 \Rightarrow 0 = 1(1 \cdot e + C) \Rightarrow C = -e$

So, (i) becomes, $x = y^2(y \cdot e^y - e)$

$\therefore \quad x(e) = e^2(e \cdot e^e - e) = e^3(e^e - 1)$

15. (d): The given equation of tangent can be rewritten as,

$$y = \frac{\lambda x}{2} - \frac{\mu}{2} \quad \text{...(i)}$$

Now, we know that, (i) touches the given hyperbola if,

$$\left(\frac{-\mu}{2}\right)^2 = \left(\frac{b}{a}\right)^2 \left(\frac{\lambda}{2}\right)^2 - b^2$$

[$\because$ Any line $y = mx + c$ touches the hyperbola $x^2/a^2 - y^2/b^2 = 1$ if $c^2 = a^2m^2 - b^2$]

$$\Rightarrow \frac{\mu^2}{4} = \frac{b^2}{a^2}\frac{\lambda^2}{4} - b^2 \Rightarrow \frac{\mu^2}{b^2} = \frac{\lambda^2}{a^2} - 4$$

$$\Rightarrow \left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2 = 4$$

16. (c): We have,

$$\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a}),\ |\hat{b}|^2 = |\vec{c} + 2(\vec{c} \times \hat{a})|^2$$

$$\Rightarrow 1 = |\vec{c}|^2 + 4|\vec{c} \times \hat{a}|^2 + 0 \qquad [\because\ \vec{c} \cdot (\vec{c} \times \hat{a}) = 0]$$

$$\Rightarrow 1 = |\vec{c}|^2 + 4|\vec{c}|^2|\hat{a}|^2 \cdot \sin^2\frac{\pi}{12}$$

$$\Rightarrow 1 = |\vec{c}|^2 + 4|\vec{c}|^2\left(\frac{1-\sqrt{3}}{2\sqrt{2}}\right)^2$$

$$\Rightarrow |\vec{c}|^2 + 4|\vec{c}|^2\left(\frac{4-2\sqrt{3}}{8}\right) = 1$$

$$\Rightarrow |\vec{c}|^2 + |\vec{c}|^2(2-\sqrt{3}) = 1$$

$$\Rightarrow |\vec{c}|^2(3-\sqrt{3}) = 1 \Rightarrow |\vec{c}|^2 = \frac{1}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$$

$$\Rightarrow |6\vec{c}|^2 = 36|\vec{c}|^2 = 36 \cdot \frac{3+\sqrt{3}}{6} = 6(3+\sqrt{3})$$

17. (a): Here, $n = 33$

Also, $3P(X = 0) = P(X = 1)$

$$\Rightarrow 3 \cdot {}^{33}C_0\, p^0 q^{33} = {}^{33}C_1\, p^1 q^{32} \Rightarrow 3q^{33} = 33pq^{32}$$

$$\Rightarrow \frac{q}{p} = 11 \quad \text{...(i)}$$

$$\text{Now, } \frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$$

$$= \frac{{}^{33}C_{15}p^{15}q^{18}}{{}^{33}C_{18}p^{18}q^{15}} - \frac{{}^{33}C_{16}p^{16}q^{17}}{{}^{33}C_{17}p^{17}q^{16}}$$

$$= \left(\frac{q}{p}\right)^3 - \left(\frac{q}{p}\right) \qquad [\because\ {}^nC_r = {}^nC_{n-r}]$$

$= (11)^3 - 11$ [Using (i)]

$= 1320$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

6				1				3				3				8				9
		1		8		6								8		7		5		
	8						1						2						1	
			8		5										1		8			
3	2						5	7				4	1						5	2
			3		2										5		7			
	4						8			1			5						9	
		9		2		1			8		4			2		6		1		
7				9				2				8				5				6
							5						1							
						9								5						
							2						6							
4				8				1				2				3				8
		6		3		8			1		2			3		1		9		
	8						6			3			8						2	
			5		9										2		1			
1	7						5	9				9	1						4	2
			4		3										4		7			
	1						2						5						8	
		4		9		1								7		8		5		
8				2				7				4				7				1

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

18. (*) : We have, $f(x) = \dfrac{\cos^{-1}\left(\dfrac{x^2-5x+6}{x^2-9}\right)}{\log_e(x^2-3x+2)}$

For domain, $x^2 - 3x + 2 > 0 \Rightarrow (x-2)(x-1) > 0$

$\Rightarrow x < 1$ or $x > 2$ ...(i)

Also, $\log_e(x^2 - 3x + 2) \neq 0$

$\Rightarrow x^2 - 3x + 2 \neq 1 \Rightarrow x^2 - 3x + 1 \neq 0$

$$\Rightarrow x \neq \left\{\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right\} \qquad \text{...(ii)}$$

$$\text{Now, } -1 \leq \frac{x^2-5x+6}{x^2-9} \leq 1$$

$$\Rightarrow -1 \leq \frac{(x-2)(x-3)}{(x-3)(x+3)} \leq 1 \Rightarrow -1 \leq \frac{x-2}{x+3} \leq 1$$

Case (I) $\dfrac{x-2}{x+3} + 1 \geq 0 \Rightarrow \dfrac{2x+1}{x+3} \geq 0$

$$\Rightarrow x \geq -\frac{1}{2} \text{ or } x < -3 \qquad \text{...(iii)}$$

Case (II) $\dfrac{x-2}{x+3} - 1 \leq 0 \Rightarrow \dfrac{-5}{x+3} \leq 0$

$\Rightarrow x + 3 > 0$

$\Rightarrow x > -3$ ...(iv)

And, $x^2 - 9 \neq 0$

$\Rightarrow x \neq \pm 3$ ...(v)

So, required domain is,

$$\left[\frac{-1}{2}, 1\right) \cup (2,3) \cup (3,\infty) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$$

19. (b): We have,

$\tan\theta \cdot \sin\theta + \tan\theta = \sin 2\theta$

$$\Rightarrow \sin\theta \cdot \frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta} - 2\sin\theta\cos\theta = 0$$

$$\Rightarrow \sin\theta\left[\left(\frac{\sin\theta+1}{\cos\theta}\right) - 2\cos\theta\right] = 0$$

$$\Rightarrow \sin\theta = 0 \text{ or } \frac{\sin\theta + 1 - 2\cos^2\theta}{\cos\theta} = 0$$

$\Rightarrow \theta = 0, \pi, -\pi$ ...(i)

or $\sin\theta + 1 - 2\cos^2\theta = 0$

$\Rightarrow \sin\theta + 1 - 2(1 - \sin^2\theta) = 0$

$\Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$

$\Rightarrow (\sin\theta + 1)(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = -1$ or $\sin\theta = \dfrac{1}{2}$

$\Rightarrow \theta = \dfrac{-\pi}{2}$ (which is not possible)

or $\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ $\quad\therefore S = \left\{0, \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \pi, -\pi\right\}$

$\Rightarrow n(S) = 5$

Now,

$$T = \sum_{\theta\in S}\cos 2\theta = \cos 0 + \cos\frac{\pi}{3} + \cos\frac{5\pi}{3} + \cos 2\pi + \cos(-2\pi)$$

$$= 1 + \frac{1}{2} + \frac{1}{2} + 1 + 1 = 4 \qquad \therefore T + n(S) = 4 + 5 = 9$$

20. (b): Only possibilities for $\Delta = \{\vee, \Rightarrow\}$

21. (31) : We have, $2f(a) - f(b) + 3f(c) + f(d) = 0$

$\Rightarrow 2f(a) + 3f(c) = f(b) - f(d)$

Also, possible values $f(a)$ can take = 0, 1, 2, 3, 4

and possible values $f(c)$ can take = 0, 1, 2, 3

$f(c)$	$f(a)$	No. of possible ways for $2f(a) + 3f(c) = f(b) - f(d)$	
0	1	7	$(\because f(b) - f(d) = 2)$
	2	5	$(\because f(b) - f(d) = 4)$
	3	3	$(\because f(b) - f(d) = 6)$
	4	2	$(\because f(b) - f(d) = 8)$
1	0	6	$(\because f(b) - f(d) = 3)$
	2	2	$(\because f(b) - f(d) = 7)$
	3	1	$(\because f(b) - f(d) = 9)$
2	0	3	$(\because f(b) - f(d) = 6)$
	1	1	$(\because f(b) - f(d) = 8)$
3	0	1	$(\because f(b) - f(d) = 9)$

Total no. of possible ways = 31.

22. (40) : The only choice for getting 5 marks is 3 correct answers and 2 incorrect answers.

Now, no. of ways in which student answers 3 questions correctly = 1

and no. of ways in which 2 questions are incorrectly answer = 2 × 2 = 4

$\therefore$ Required no. of ways = ${}^5C_3\,(1)(4) = 40$

23. (8) : We have, $A = \left(\dfrac{3}{\sqrt{a}}, \sqrt{a}\right), a > 0$

Image of A in y-axis is $\left(\dfrac{-3}{\sqrt{a}}, \sqrt{a}\right)$

So, coordinates of $B = \left(\dfrac{-3}{\sqrt{a}}, \sqrt{a}\right)$

Similarly, coordinates of $C = \left(\dfrac{-3}{\sqrt{a}}, -\sqrt{a}\right)$

Now, maximum area of $\Delta ACD = 12$

$$\Rightarrow \max.\left[\frac{1}{2}\begin{vmatrix} 3/\sqrt{a} & \sqrt{a} & 1\\ -3/\sqrt{a} & -\sqrt{a} & 1\\ 3\cos\theta & a\sin\theta & 1\end{vmatrix}\right] = \pm 12$$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \max.\left[\frac{1}{2}\begin{vmatrix} 0 & 0 & 2\\ -3/\sqrt{a} & -\sqrt{a} & 1\\ 3\cos\theta & a\sin\theta & 1\end{vmatrix}\right] = \pm 12$$

$$\Rightarrow \max.[(-3\sqrt{a}\sin\theta + 3\sqrt{a}\cos\theta)] = \pm 12$$

$$\Rightarrow \max.[\sqrt{a}(\cos\theta - \sin\theta)] = \pm 4$$

Maximum value of $\cos\theta - \sin\theta = \sqrt{1^2+1^2} = \sqrt{2}$

$\therefore \sqrt{a}\sqrt{2} = \pm 4$

$\Rightarrow 2a = 16 \Rightarrow a = 8$

24. (84) : We have, $\dfrac{x-7}{3} = \dfrac{y-1}{-1} = \dfrac{z+2}{1} = \lambda(\text{say})$

and $\dfrac{x}{2} = \dfrac{y-7}{3} = \dfrac{z}{1} = \mu(\text{say})$

Let $A = (3\lambda + 7, 1 - \lambda, \lambda - 2)$ and $B = (2\mu, 3\mu + 7, \mu)$

$\therefore$ Dr's of $AB = < 3\lambda - 2\mu + 7, -\lambda - 3\mu - 6, \lambda - \mu - 2 >$

Now, $\dfrac{3\lambda - 2\mu + 7}{1} = \dfrac{-\lambda - 3\mu - 6}{-4} = \dfrac{\lambda - \mu - 2}{2}$

$\Rightarrow 4(3\lambda - 2\mu + 7) = \lambda + 3\mu + 6$

$\Rightarrow \mu - \lambda = 2$ ...(i)

and $2(3\lambda - 2\mu + 7) = \lambda - \mu - 2$

and $5\lambda - 3\mu + 16 = 0$ ...(ii)

Solving (i) and (ii), we get $\lambda = -5$ and $\mu = -3$

$\therefore A = (-8, 6, -7), B = (-6, -2, -3) \Rightarrow (AB)^2 = 84$

25. (7)

26. (1) : We have,

$$f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta) f(t)dt$$

$$= \sin\theta\left\{1 + \int_{-\pi/2}^{\pi/2} f(t)dt\right\} + \cos\theta \int_{-\pi/2}^{\pi/2} t\, f(t)dt$$

$\Rightarrow f(\theta) = a\sin\theta + b\cos\theta$ ...(i)

where, $a = 1 + \displaystyle\int_{-\pi/2}^{\pi/2} f(t)dt$

$\Rightarrow a = 1 + \displaystyle\int_{-\pi/2}^{\pi/2} (a\sin t + b\cos t)dt$ ...(1)

$\Rightarrow a = 1 + \displaystyle\int_{-\pi/2}^{\pi/2} (-a\sin t + b\cos t)dt$ ...(2)

Adding (1) and (2), we get

$$2a = 2 + \int_{-\pi/2}^{\pi/2} 2b\cos t\; dt$$

$a = 1 + 2b$ ...(ii)

Also, $b = \displaystyle\int_{-\pi/2}^{\pi/2} t\, f(t)dt = \int_{-\pi/2}^{\pi/2} t\{b\cos t + a\sin t\}dt$ ...(3)

$\Rightarrow b = \displaystyle\int_{-\pi/2}^{\pi/2} (at\sin t - tb\cos t)dt$ ...(4)

Adding (3) and (4), we get

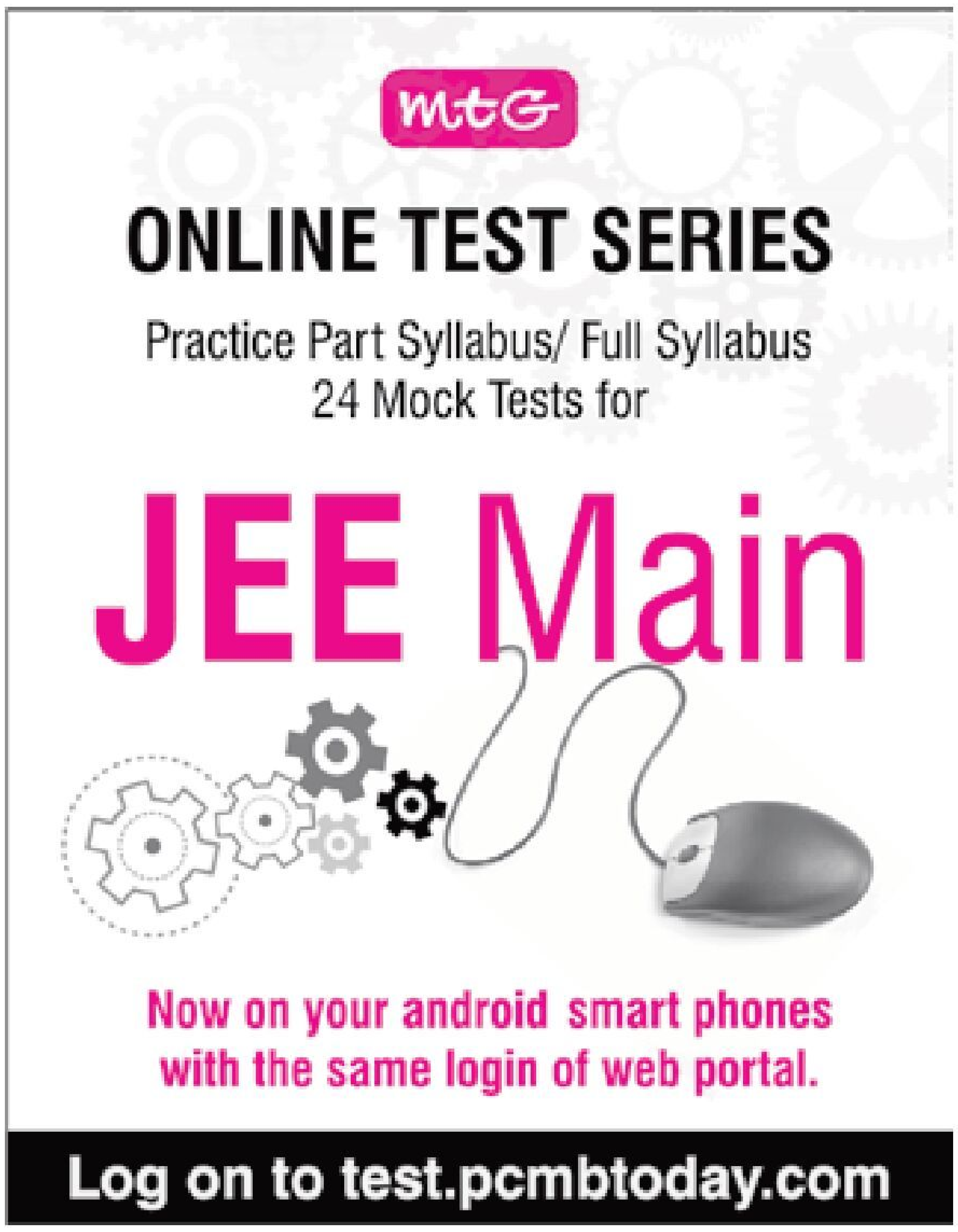
$$2b = \int_{-\pi/2}^{\pi/2} 2at\,\sin t\,dt = 4a$$

$\Rightarrow b = 2a$ ...(iii)

From (ii) and (iii), we get

$a = -\dfrac{1}{3}, b = -\dfrac{2}{3}, \quad \therefore f(\theta) = -\dfrac{1}{3}\{\sin\theta + 2\cos\theta\}$

$$\Rightarrow \int_0^{\pi/2} f(\theta)d\theta = -\frac{1}{3}\int_0^{\pi/2} (\sin\theta + 2\cos\theta)\,d\theta$$



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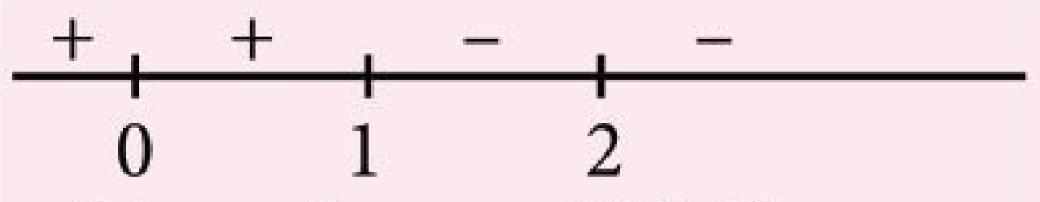
$$= -\frac{1}{3}\times 3 = -1 \quad \Rightarrow \quad \left|\int_0^{\pi/2} f(\theta)d\theta\right| = 1$$

27. (34) : Let $f(x) = \frac{9-x^2}{5-x} = \frac{x^2-9}{x-5}$

$$\Rightarrow \quad f'(x) = \frac{(x-5)(2x)-(x^2-9)(1)}{(x-5)^2} = \frac{x^2-10x+9}{(x-5)^2}$$

$$= \frac{(x-1)(x-9)}{(x-5)^2}$$



(sign scheme of $f'(x)$)

So, $\alpha = f(1) = \frac{8}{4} = 2$

Now, $f(0) = \frac{9}{5}$, $f(2) = \frac{5}{3}$

$\therefore \quad \beta = f(2) = \frac{5}{3}$

Now, $\int_{\beta-8/3}^{2\alpha-1} \text{Max}\left\{\frac{9-x^2}{5-x}, x\right\}dx = \int_{-1}^{3} \text{Max}\left\{\frac{9-x^2}{5-x}, x\right\}dx$

$$= \int_{-1}^{9/5}\left(\frac{9-x^2}{5-x}\right)dx + \int_{9/5}^{3} x\,dx$$

$$= \int_{-1}^{9/5}\left(\frac{16}{x-5}+5+x\right)dx + \left[\frac{x^2}{2}\right]_{9/5}^{3}$$

$$= \left[16\log|x-5| + 5x + \frac{x^2}{2}\right]_{-1}^{9/5} + \left(\frac{9}{2}-\frac{81}{50}\right)$$

$$= \left[\left(9+\frac{81}{50}\right) - \left(-5+\frac{1}{2}\right) + 16\left\{\log\left|\frac{16}{5}\right| - \log|6|\right\}\right] + \frac{72}{25}$$

$$= 18 + 16\log\left(\frac{8}{15}\right)$$

$\therefore \quad \alpha_1 = 18, \alpha_2 = 16, \qquad \alpha_1 + \alpha_2 = 34$

28. (2929) : Tangent to the given parabola $y^2 = 4x$ is

$$y = mx + \frac{1}{m} \qquad \text{...(i)}$$

(i) passes through (α, β) $\quad\therefore\quad \beta = \alpha m + \frac{1}{m}$

$\Rightarrow \alpha m^2 - \beta m + 1 = 0 \qquad$ ...(ii)

$\Rightarrow m_1 + m_2 = \frac{\beta}{\alpha}, \; m_1 m_2 = \frac{1}{\alpha}$

But $m_1 = 4\,m_2 \quad \Rightarrow \quad 5m_2 = \frac{\beta}{\alpha}, 4m_2^2 = \frac{1}{\alpha}$

$\Rightarrow \quad \frac{\beta^2}{25\alpha^2} = \frac{1}{4\alpha} \Rightarrow 4\beta^2 = 25\alpha \qquad$ ...(ii)

Also, (α, β) lies on ellipse $25x^2 + 4y^2 = 1$

$25\alpha^2 + 4\beta^2 = 1 \qquad$ ...(iii)

From (ii) and (iii), we get

$25\alpha^2 + 25\alpha - 1 = 0 \qquad$ ...(iv)

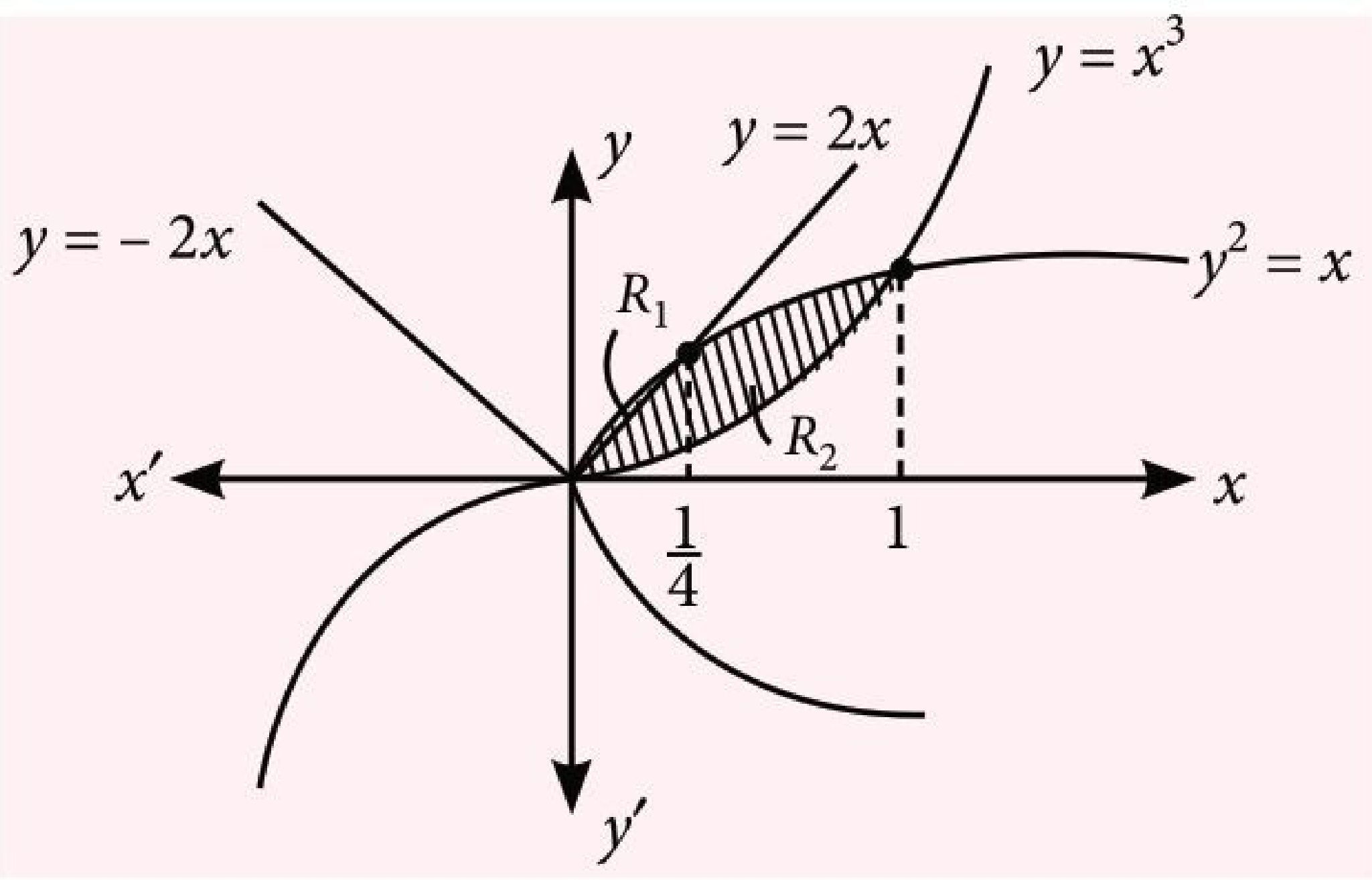
Now, $(10\alpha + 5)^2 + (16\beta^2 + 50)^2 = (10\alpha + 5)^2 + (4(25\alpha) + 50)^2$

$= 25(2\alpha + 1)^2 + 50^2(2\alpha + 1)^2$

$= 2525(2\alpha + 1)^2 = 2525\,(4\alpha^2 + 4\alpha + 1)$

$$= 2525\left\{\frac{4}{25}+1\right\} = 101\times 29 = 2929$$

29. (19) : Given, $y = x^3, y^2 = x$ and $y = 2|x|$



$$R_1 = \int_0^{\frac{1}{4}}\left(\sqrt{x}-2x\right)dx = \left[\frac{2}{3}x^{3/2} - x^2\right]_0^{\frac{1}{4}} = \frac{1}{48}$$

$$R_2 = \int_0^{\frac{1}{4}}(2x - x^3)\,dx + \int_{\frac{1}{4}}^{1}\left(\sqrt{x} - x^3\right)dx$$

$$= \left[x^2 - \frac{x^4}{4}\right]_0^{\frac{1}{4}} + \left[\frac{2}{3}x^{3/2} - \frac{x^4}{4}\right]_{\frac{1}{4}}^{1} = \frac{19}{48}$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{19}{48}\times\frac{48}{1} = 19$$

30. (2) : We have, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix}$

$$= -a\hat{i} - \hat{j} + (a-1)\hat{k}$$

Also, $\vec{a}_1 - \vec{a}_2 = -\hat{i} + \hat{j} + \hat{k}$

$\therefore$ Shortest distance $= \left|\frac{(\vec{a}_1 - \vec{a}_2)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$\Rightarrow \quad \sqrt{\frac{2}{3}} = \left|\frac{a-1+a-1}{\sqrt{a^2+1+(a-1)^2}}\right|$$

$$\Rightarrow \quad \frac{2}{3} = \frac{(2a-2)^2}{a^2+1+a^2+1-2a} \quad \Rightarrow \quad \frac{1}{3} = \frac{2(a^2+1-2a)}{2a^2+2-2a}$$

$\Rightarrow \; 2a^2 + 2 - 2a = 6a^2 + 6 - 12a \Rightarrow 2a^2 - 5a + 2 = 0$

$\Rightarrow \; (a - 2)(2a - 1) = 0 \;\; \Rightarrow \;\; a = 2, \frac{1}{2}$

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JEE 2022 PRACTICE PAPER ADVANCED

Exam on 28th August

PAPER-1

SECTION 1

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*

1. A line $y = mx + 1$ intersects the circle $(x - 3)^2 + (y + 2)^2 = 25$ at points P and Q. If the midpoint of the line segment PQ has x-coordinate $-3/5$, then which one of the following options is correct?

(a) $6 \le m < 8$ (b) $2 \le m < 4$
(c) $4 \le m < 6$ (d) $-3 \le m < -1$

2. Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$,

where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2×2 identity matrix.
If α^* is the minimum of the set $\{\alpha(\theta) : \theta \in [0, 2\pi)\}$ and β^* is the minimum of the set $\{\beta(\theta) : \theta \in [0, 2\pi)\}$, then the value of $\alpha^* + \beta^*$ is

(a) $\frac{-31}{16}$ (b) $\frac{-29}{16}$
(c) $\frac{-37}{16}$ (d) $\frac{-17}{16}$

3. Let complex numbers α and $\frac{1}{\bar{\alpha}}$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha| =$

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{3}$

4. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$

(a) on the left of $x = c$
(b) on the right of $x = c$
(c) at no point
(d) at all points.

SECTION 2

- *This section contains THREE (03) questions stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 5 and 6

Let $f(x) = \min\left\{x^{\frac{2}{3}}, \sqrt{2-x^2}\right\}$ and

$g(x) = \max\left\{x^{\frac{2}{3}}, \sqrt{2-x^2}\right\}, |x| \le \sqrt{2}$

5. The number of points at which $f(x)$ is not differentiable is

6. The number of points at which $g(x)$ is not differentiable is

Question Stem for Question Nos. 7 and 8

Tangent at a point P_1 [other than (0, 0)] on the curve $y = x^3$ meets the curve again at P_2. The tangent at P_2 meets the curve again at P_3 and so on.

7. If P_1 has coordinates (1, 1), then the sum $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{x_n}$ (where $x_1, x_2, \ldots$ are abscissas of $P_1, P_2, \ldots$, respectively)

8. If P_1 has coordinates (1, 1), then the sum $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{y_n}$ (where $y_1, y_2, \ldots$ are ordinates of $P_1, P_2, \ldots$ respectively)

Question Stem for Question Nos. 9 and 10

A 3×3 determinant has entries either 1 or -1. Let S_3 be set of all determinants which contain product of elements of any row or any column as -1.

For example, $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix}$ is an element of the set S_3.

9. Number of elements of the set S_3 is

10. Number of elements of the set S_n is $a^{(n-1)^b}$, then $a + b =$ _______.

SECTION 3

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

11. Define the collections $\{E_1, E_2, E_3, ...\}$ of ellipses and $\{R_1, R_2, R_3, ...\}$ of rectangles as follows :

$E_1 : \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1;$

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1 :

E_n : ellipse $\dfrac{x^2}{a_n^2} + \dfrac{y^2}{b_n^2} = 1$ of largest area inscribed in R_{n-1}, $n > 1$; R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n, $n > 1$.

Then which of the following options is/are correct?

(a) The eccentricities of E_{18} and E_{19} are NOT equal.

(b) The length of latus rectum of E_9 is 1/6.

(c) $\sum\limits_{n=1}^{N}$ (area of R_n) < 24, for each positive integer N.

(d) The distance of a focus from the centre in E_9 is $\dfrac{\sqrt{5}}{32}$.

12. In a non-right-angled triangle ΔPQR, let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}$, $q = 1$, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct?

(a) Length of $RS = \dfrac{\sqrt{7}}{2}$

(b) Area of $\Delta SOE = \dfrac{\sqrt{3}}{12}$

(c) Radius of incircle of $\Delta PQR = \dfrac{\sqrt{3}}{2}(2-\sqrt{3})$

(d) Length of $OE = 1/6$

13. Let $f : R \to R$ be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \le x < 1; \\ (2/3)x^3 - 4x^2 + 7x - (8/3), & 1 \le x < 3; \\ (x-2)\log_e(x-2) - x + (10/3), & x \ge 3. \end{cases}$$

Then which of the following options is/are correct?

(a) f' is NOT differentiable at $x = 1$

(b) f is increasing on $(-\infty, 0)$

(c) f is onto

(d) f' has a local maximum at $x = 1$

14. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define $a_n = \dfrac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \ge 1$, $b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \ge 2$.

Then which of the following options is/are correct?

(a) $\sum\limits_{n=1}^{\infty} \dfrac{a_n}{10^n} = \dfrac{10}{89}$

(b) $b_n = \alpha^n + \beta^n$ for all $n \ge 1$

(c) $a_1 + a_2 + ... + a_n = a_{n+2} - 1$ for all $n \ge 1$

(d) $\sum\limits_{n=1}^{\infty} \dfrac{b_n}{10^n} = \dfrac{8}{89}$

15. Let L_1 and L_2 denote the lines

$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k})$, $\lambda \in R$ and

$\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k})$, $\mu \in R$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3?

(a) $\vec{r} = \dfrac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, $t \in R$

(b) $\vec{r} = \dfrac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, $t \in R$

(c) $\vec{r} = \dfrac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, $t \in R$

(d) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k})$, $t \in R$

16. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities 3/10, 3/10 and 4/10 respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

(a) Probability that the chosen ball is green, given that the selected bag is B_3, equals 3/8.

(b) Probability that the selected bag is B_3, given that the chosen ball is green, equals 5/13.

(c) Probability that the chosen ball is green equals 39/80

(d) Probability that the selected bag is B_3, and the chosen ball is green equals 3/10

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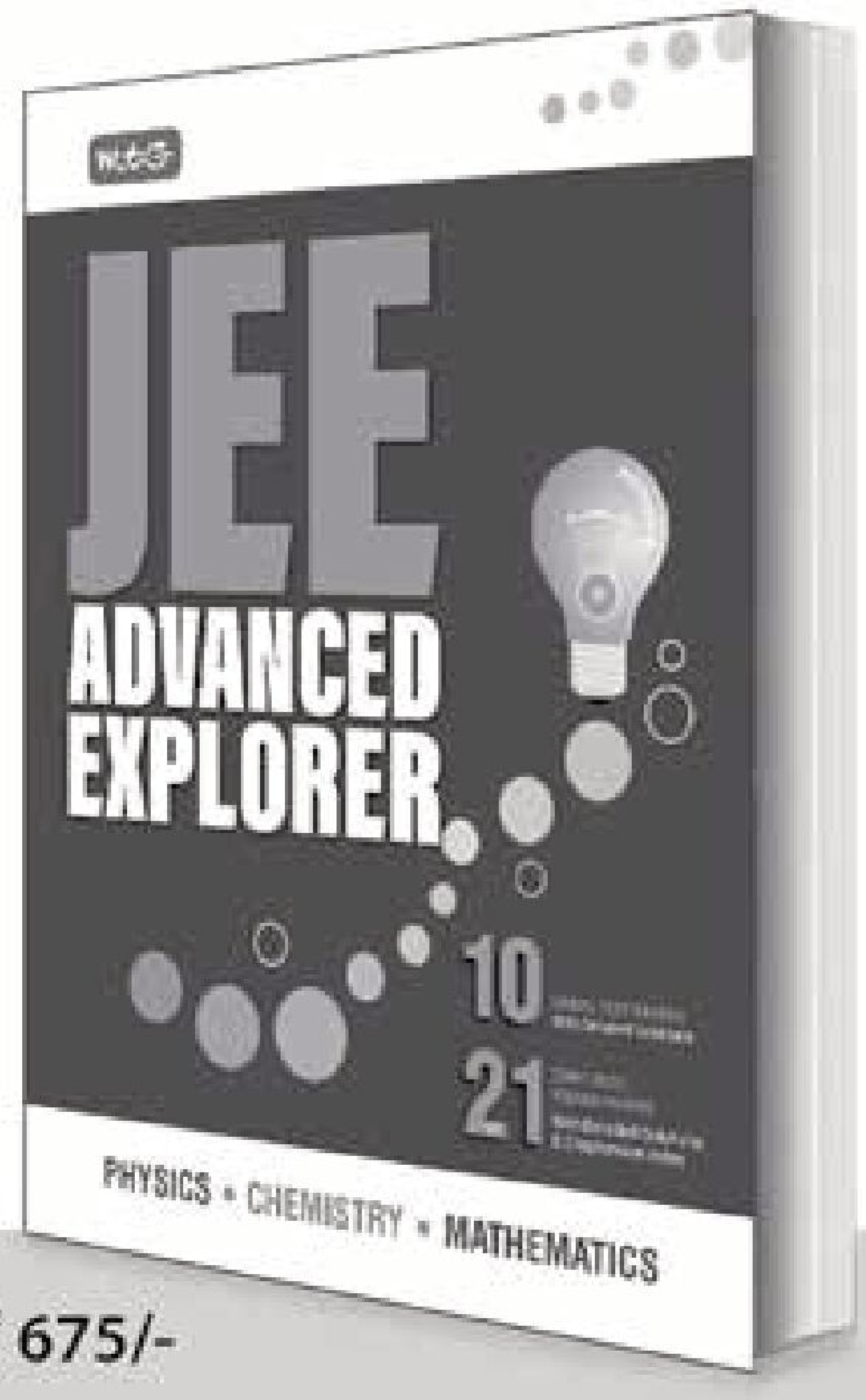
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SECTION 4

- *This section contains THREE (03) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*

17. Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\{|a + b\omega + c\omega^2|^2 : a, b, c$ distinct non-zero integers$\}$ equals _____.

18. Find the sum of all divisors of the sum of all integral values of α for which one root of the equation $(\alpha - 5)x^2 - 2\alpha x + \alpha - 4 = 0$ is smaller than 1 and other is greater than 2.

19. Five persons A, B, C, D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is __.

PAPER-2

SECTION 1

- *This section contains SIX (06) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*

1. The orthogonal trajectories of $\dfrac{x^2}{a^2}+\dfrac{y^2}{a^2+\lambda}=1$ (where λ is an arbitrary parameter) is given by $x^{k_1}+y^{k_2}=k_3(a^{k_4}\log x+1)$ whenever the orthogonal trajectory passes through (1, 1) then

(a) $k_1 = k_2$ (b) $k_1 = k_2 = k_3$

(c) $k_1 = k_2 = k_3 = k_4$ (d) $k_1^5 = 16k_2$

2. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, then which of the following statement(s) is/are true ?

(a) $\vec{a}\times(\vec{b}\times\vec{c}), \vec{b}\times(\vec{c}\times\vec{a}), \vec{c}\times(\vec{a}\times\vec{b})$ form a right handed system.

(b) $\vec{c}, (\vec{a}\times\vec{b})\times\vec{c}, \vec{a}\times\vec{b}$ form a right handed system.

(c) $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}<0$ if $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

(d) $\dfrac{(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})}{(\vec{b}\times\vec{c})\cdot(\vec{a}\times\vec{c})}=-1$ if $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

3. The plane $x - 2y + 7z + 21 = 0$

(a) contains the line $\dfrac{x+1}{-3}=\dfrac{y-3}{2}=\dfrac{z+2}{1}$.

(b) contains the point $(0, 7, -1)$.

(c) is perpendicular to the line $\dfrac{x}{1}=\dfrac{y}{-2}=\dfrac{z}{7}$.

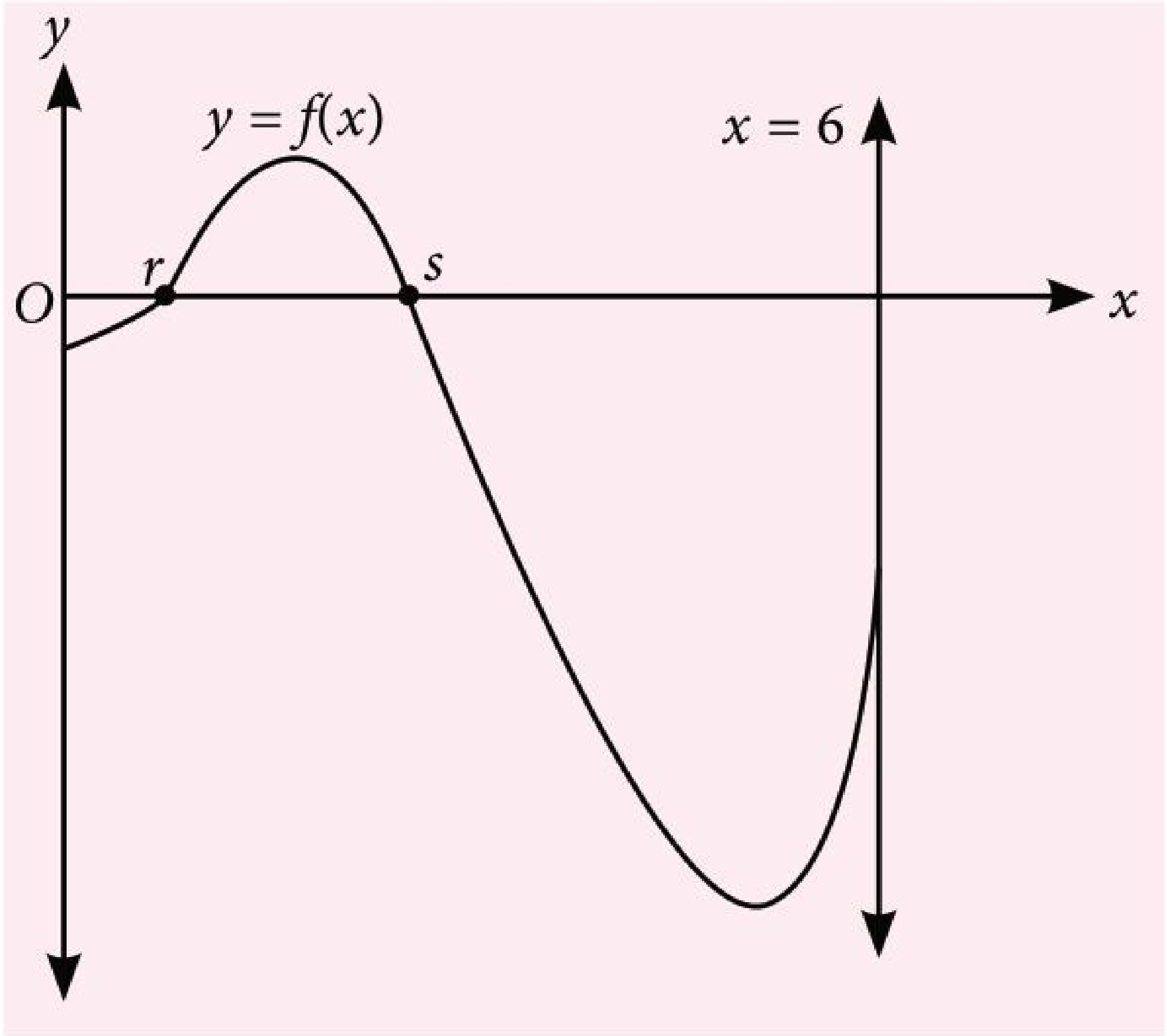
(d) is parallel to the plane $x - 2y + 7z = 0$.

4. Let $f'(x)=\dfrac{192x^3}{2+\sin^4 \pi x}$ for all $x \in R$ with $f\left(\dfrac{1}{2}\right)=0$. If $m \le \displaystyle\int_{1/2}^{1} f(x)dx \le M$, then the possible values of m and M are

(a) $m = 13, M = 24$ (b) $m=\dfrac{1}{4}, M=\dfrac{1}{2}$

(c) $m = -11, M = 10$ (d) $m = 1, M = 12$

5. The graph of $y = f(x)$ is shown with roots r and s $(r < s)$. Area bounded by the graph of $f(x)$, x-axis, $x = 0$ and $x = 6$ over the intervals $[0, r]$ and $[r, s]$ and $[s, 6]$ are $\dfrac{2}{5}$, 2 and 12 respectively.



If $m=\displaystyle\int_s^0 f(x)dx, p=\int_r^6 f(x)dx, q=\left|\int_0^6 f(x)dx\right|, n=\int_0^6 |f(x)|dx,$ then

(a) $p < m$ (b) $m < 1$

(c) $q < 11$ (d) $n > 11$

6. If $f(x) = \dfrac{x}{2} - 1$, then on the interval $[0, \pi]$

(a) $\tan f(x)$ and $\dfrac{1}{f(x)}$ are continuous

(b) $\tan f(x)$ and $\dfrac{1}{f(x)}$ are discontinuous

(c) $\tan f(x)$ and $\dfrac{1}{f^{-1}(x)}$ are continuous

(d) $\tan f(x)$ is continuous but $\dfrac{1}{f(x)}$ is not continuous

SECTION 2

- *This section contains THREE (03) question stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 7 and 8

All small squares are of the same size. A shortest route from A to C is defined as a route consisting 8 horizontal steps (H) and 5 vertical steps (V). Since any shortest route is a typical arrangement of $8H$ and $5V$, the number of shortest routes is $\frac{13!}{5!\,8!}$.

7. The number of shortest routes through the junction P is

8. The number of shortest routes which go following street PQ must be

Question Stem for Question Nos. 9 and 10

$\lim\limits_{x\to 0}\frac{1-\cos(f(x))}{(f(x))^2}=\frac{1}{2}$ if $\lim\limits_{x\to 0} f(x)=0$ and α, β, γ are the real roots of $ax^3 + bx^2 + cx + d = 0$, then

9. $\lim\limits_{x\to 0}\dfrac{1-\cos 1\cdot\cos(\cos x)-\sin 1\cdot\sin(\cos x)}{x^4}=$

10.

$$\lim_{x\to 0}\frac{1-\cos^2(\sin x)-\cos(\sin^2 x)+\cos^2(\sin x)\cos(\sin^2 x)}{x^6}=$$

Question Stem for Question Nos. 11 and 12

Consider the data 1, 2, 3, ..., 103, 104

11. Quartile deviation of the given data is

12. Mean deviation from the median of the given data is

SECTION 3

- *This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*

Paragraph for Q. 13 and Q. 14

Box 1 contains three tickets bearing numbers 1, 2, 3; box 2 contains five tickets bearing numbers 3, 4, 5, 6, 7; and box 3 contains seven tickets bearing numbers 3, 4, 5, 6, 7, 8, 9. A ticket is drawn from each of the boxes. Let x_i be the number on the tickets drawn from the i^{th} box, $i = 1, 2, 3$.

13. The probability that $x_1 + x_2 + x_3$ is odd, is

(a) $\frac{29}{105}$ (b) $\frac{53}{105}$

(c) $\frac{57}{105}$ (d) $\frac{1}{2}$

14. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(a) $\frac{9}{105}$ (b) $\frac{10}{105}$

(c) $\frac{11}{105}$ (d) $\frac{7}{105}$

Paragraph for Q. 15 and Q. 16

Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science and 4 in all the three subjects.

15. The number of students passed in English and Mathematics but not in Science is

(a) 3 (b) 2 (c) 4 (d) 5

16. The number of students passed in Mathematics only is

(a) 5 (b) 4 (c) 3 (d) 2

SECTION 4

- *This section contains THREE (03) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*

17. If $y = (1 + 1/x)^x$ then $\dfrac{2\sqrt{y_2(2)+1/8}}{(\log 3/2-1/3)}$ is equal to

18. If the area of the triangle included between the axes and any tangent to the curve $x^n y = a^n$ is constant, then n is equal to

19. If $g(x) = \left(4\cos^4 x-2\cos 2x-\frac{1}{2}\cos 4x-x^7\right)^{1/7}$, then the value of $\dfrac{g(g(100))}{100}$ is equal to

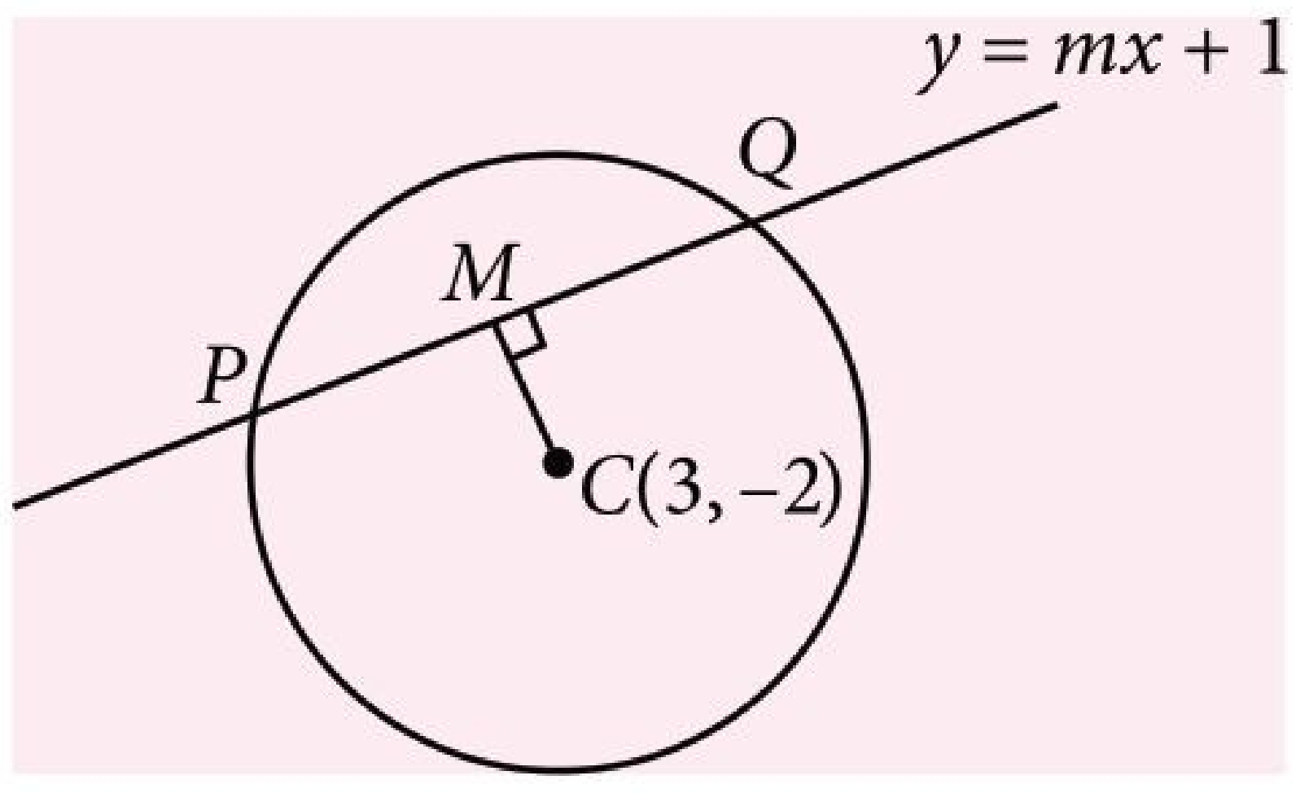
MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1. (c) **2.** (b) **3.** (a) **4.** (b) **5.** (b)
6. (b) **7.** (a) **8.** (a, c) **9.** (a,b,c) **10.** (a,b,c)
11. (a, b, c) **12.** (b) **13.** (a, d) **14.** (c) **15.** (d)
16. (a) **17.** (360) **18.** (2) **19.** (3) **20.** (6)

SOLUTIONS

PAPER-1

1. (b) :



Let M be the midpoint of PQ.

$y = mx + 1 \Rightarrow y = m(-3/5) + 1$

$$\left[\because x\text{-coordinate of } M = \frac{-3}{5} \text{ (Given)}\right]$$

$$\Rightarrow y = 1 - \frac{3m}{5}$$

$\therefore$ Coordinates of M are $\left(-\frac{3}{5},\ 1-\frac{3m}{5}\right)$

Let C be the centre of the circle.

Since $CM \perp PQ$

$$\therefore \left(\frac{1-\frac{3m}{5}-(-2)}{-\frac{3}{5}-3}\right)m = -1 \Rightarrow \left(3-\frac{3m}{5}\right)m = \frac{18}{5}$$

$\Rightarrow (5 - m)m = 6 \Rightarrow m^2 - 5m + 6 = 0$

$\Rightarrow (m - 2)(m - 3) = 0 \Rightarrow m = 2, 3$

Thus, $2 \le m < 4$.

2. (b) : Given, $M = \alpha I + \beta M^{-1} \Rightarrow M^2 = \alpha M + \beta I$ ...(i)

$$\text{For } M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix}$$

$$M^2 = \begin{bmatrix} S^8+(1+C^2)(-1-S^2) & (-1-S^2)(S^4+C^4) \\ (1+C^2)(S^4+C^4) & C^8+(1+C^2)(-1-S^2) \end{bmatrix}, \quad \text{....(ii)}$$

where $S = \sin\theta$, $C = \cos\theta$

Substituting (ii) in (i), we get

$$\begin{bmatrix} S^8+(1+C^2)(-1-S^2) & (-1-S^2)(S^4+C^4) \\ (1+C^2)(S^4+C^4) & C^8+(1+C^2)(-1-S^2) \end{bmatrix}$$

$$= \alpha\begin{bmatrix} S^4 & -1-S^2 \\ 1+C^2 & C^4 \end{bmatrix} + \beta\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha S^4+\beta & \alpha(-1-S^2) \\ \alpha(1+C^2) & \alpha C^4+\beta \end{bmatrix}$$

On comparing the corresponding elements, we have $\alpha = S^4 + C^4$ and $\alpha C^4 + \beta = C^8 + (1 + C^2)(-1 - S^2)$

$\Rightarrow (S^4+C^4)C^4+\beta = C^8+(1+C^2)(-1-S^2)$

$\therefore \beta = -S^4C^4 - (1 + C^2)(1 + S^2)$

Now, $\alpha = \sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

$$= 1 - \frac{1}{2}\sin^2 2\theta \ge 1 - \frac{1}{2} = \frac{1}{2}$$

$\beta = -[(\sin^2\theta\cos^2\theta)^2 + 2 + \sin^2\theta\cos^2\theta]$

$\quad = -[u^2 + 2 + u]$,

where $u = \sin^2\theta\cos^2\theta = \frac{1}{4}\sin^2 2\theta \in [0, 1/4]$

Thus, $\beta_{\min}$ attained at $u = 1/4$.

$$\therefore \beta_{\min} = -\left[\frac{1}{16}+2+\frac{1}{4}\right] = -\frac{37}{16}$$

$$\text{Hence, } \alpha^* + \beta^* = \frac{1}{2} - \frac{37}{16} = -\frac{29}{16}$$

3. (c) : (Using $z\bar{z} = |z|^2$)

α and $\frac{1}{\bar{\alpha}}$ satisfy $|\alpha - z_0|^2 = r^2$ and $\left|\frac{1}{\bar{\alpha}} - z_0\right|^2 = 4r^2$

We have $|\alpha|^2 - z_0\bar{\alpha} - \alpha\bar{z}_0 + |z_0|^2 = r^2$

and $\frac{|\alpha|^2}{|\alpha|^4} - \frac{z_0\bar{\alpha}}{|\alpha|^2} - \frac{\bar{z}_0\alpha}{|\alpha|^2} + |z_0|^2 = 4r^2$

$\Rightarrow 1 - z_0\bar{\alpha} - \bar{z}_0\alpha + |\alpha|^2|z_0|^2 = 4r^2|\alpha|^2$

Using the first equation

$(|\alpha|^2 - 1) + |z_0|^2\{1 - |\alpha|^2\} = r^2(1 - 4|\alpha|^2)$

$\Rightarrow (|\alpha|^2 - 1)\{1 - |z_0|^2\} = r^2\{1 - 4|\alpha|^2\}$

Using $|z_0|^2 = \frac{r^2+2}{2}$, we get

$$(|\alpha|^2 - 1)\left\{1 - \frac{r^2+2}{2}\right\} = r^2\{1 - 4|\alpha|^2\}$$

$$\Rightarrow (|\alpha|^2 - 1)\left(-\frac{r^2}{2}\right) = r^2\{1 - 4|\alpha|^2\}$$

As $r \ne 0$, we have $|\alpha|^2 - 1 = 8|\alpha^2| - 2$

$$\Rightarrow 7|\alpha|^2 = 1 \quad \therefore |\alpha| = \frac{1}{\sqrt{7}}$$

4. (a) : Equation of tangent to $y = e^x$ at $x = c$ is

$y - e^c = e^c(x - c) \Rightarrow e^c x - y = e^c(c - 1)$ ...(1)

Equation of line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$ is

$$y - e^{c-1} = \frac{e^{c+1} - e^{c-1}}{2}(x - c + 1)$$

$$y - e^{c-1} = \frac{e^{c-1}(e^2 - 1)}{2}(x - c + 1)$$

$$\frac{e^c(e-e^{-1})}{2}x-y=e^{c-1}\left[\frac{(c-1)(e^2-1)}{2}-1\right] \quad \text{.... (2)}$$

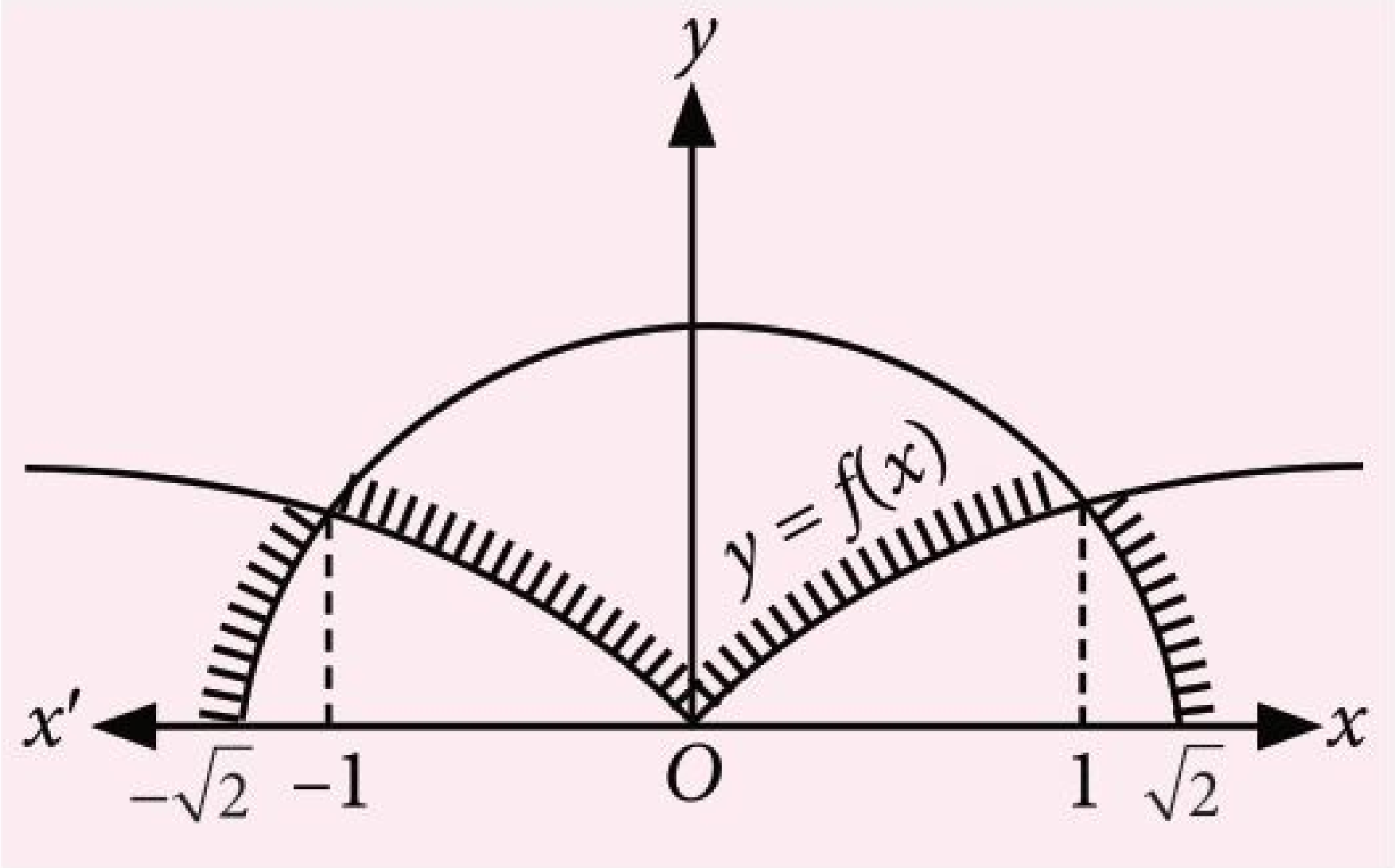
Eliminating y from (1) and (2), we get

$$e^c\left(\frac{e-e^{-1}}{2}-1\right)x=e^c\left[\frac{(c-1)(e-e^{-1})}{2}+\frac{1}{e}-(c-1)\right]$$

or $$x=\frac{(c-1)\left(\frac{e-e^{-1}}{2}-1+\frac{1}{e}\right)}{\left(\frac{e-e^{-1}}{2}-1\right)}<c$$

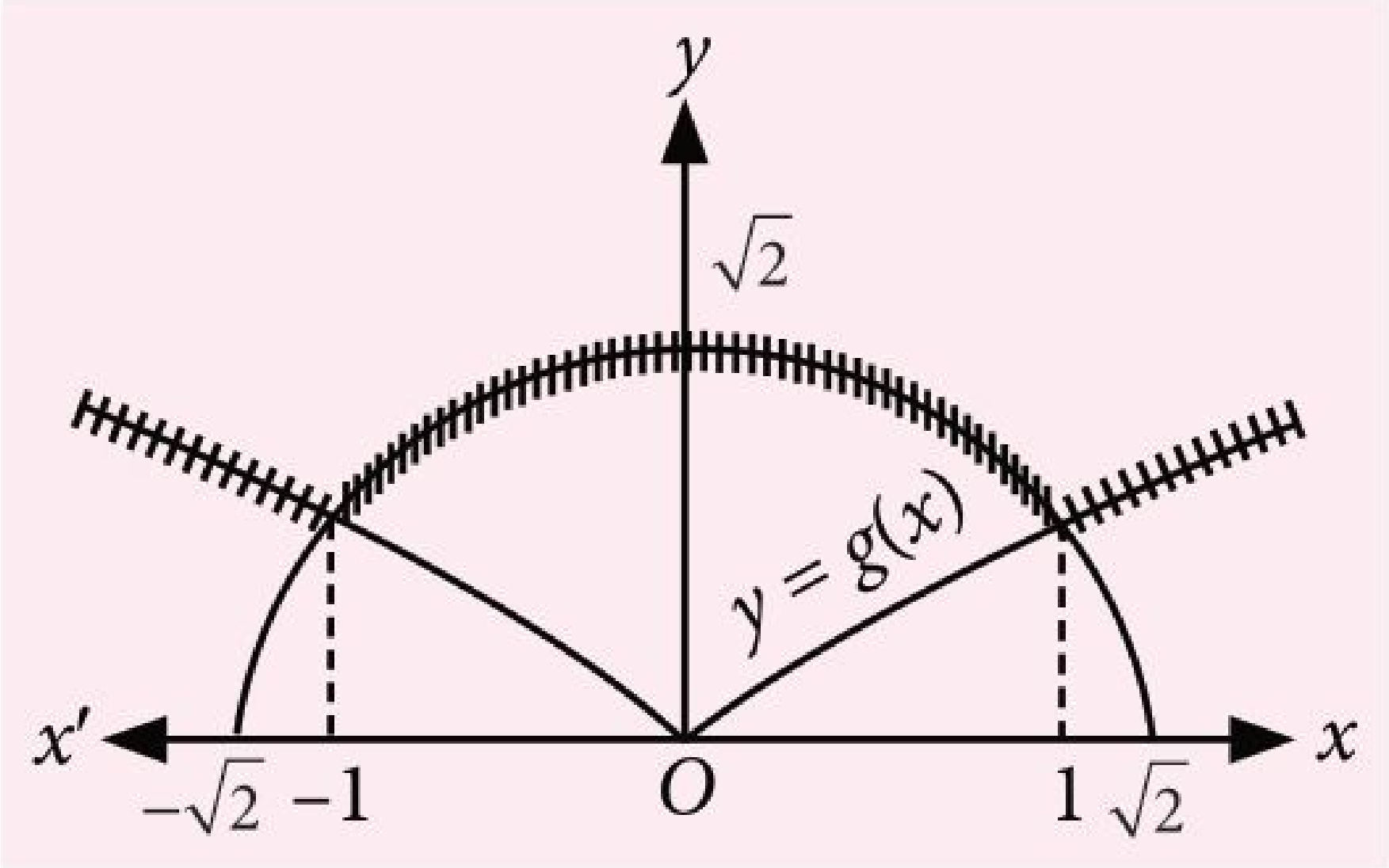
∴ Tangent will intersect the line to the left of c.

5. (3) : $$f(x)=\begin{cases}\sqrt{2-x^2}, & \text{in } \left[-\sqrt{2},-1\right]\\ x^{\frac{2}{3}}, & \text{in } [-1,1]\\ \sqrt{2-x^2}, & \text{in } \left[1,\sqrt{2}\right]\end{cases}$$



From the graph we can see that $f(x)$ is not differentiable at $x = 0, \pm 1$

6. (2) : $$g(x)=\begin{cases}x^{\frac{2}{3}} & \text{in } \left[-\sqrt{2},\ -1\right]\\ \sqrt{2-x^2} & \text{in } [-1,\ 1]\\ x^{\frac{2}{3}} & \text{in } \left[1,\sqrt{2}\right]\end{cases}$$



$y = f(x)$ is the continuous curve.
$y = g(x)$ is the curve
$g(x)$ is not differentiable at $x = \pm 1$.

7. (0.67) : Let P_1 (t_1, t_1^3) be a point on the curve $y = x^3$. Therefore $\left.\frac{dy}{dx}\right|_{(t_1,t_1^3)}=3t_1^2$

Tangent at P_1 is $y - t_1^3 = 3t_1^2(x - t_1)$ ...(i)

The intersection of (i) and $y = x^3$ is

$x^3-t_1^3=3t_1^2(x-t_1)$

or $(x - t_1)(x^2 + xt_1 + t_1^2) - 3t_1^2(x - t_1) = 0$

or $(x - t_1)^2(x + 2t_1) = 0$

If $P_2(t_2, t_2^3)$, then

$(t_2 - t_1)^2(t_2 + 2t_1) = 0 \;\therefore\; t_2 = -2t_1 \; (t_2 \neq t_1)$

Similarly, the tangent at P_2 will meet the curve at the point $P_3(t_3, t_3^3)$ when $t_3 = -2t_2 = 4t_1$ and so on.

The abscissas of $P_1, P_2, \ldots, P_n$ are

$t_1, -2t_1, 4t_1, \ldots, (-2)^{n-1}t_1$

These are in G.P. Therefore,

$$\frac{t_2}{t_1}=\frac{t_3}{t_2}=\frac{t_4}{t_3}=\ldots=-2\,(r\text{ say})$$

∴ $t_2 = t_1r$, $t_3 = t_2r$ and $t_4 = t_3r$

If $x_1 = 1$, then $x_2 = -2$, $x_3 = 4$, ... then

$\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{x_n}=$ sum of infinite G.P. with common ratio $(-1/2)$ and first term = 1

$$=\frac{1}{1-\left(-\frac{1}{2}\right)}=\frac{2}{3}$$

8. (0.89) : Then $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{y_n}=$ sum of infinite G.P. with common ratio $(-1/8)$ and first term = 1

$$=\frac{1}{1-\left(-\frac{1}{8}\right)}=\frac{8}{9}$$

(9-10) : 9. (16) 10. (4)

For S_n, $a_{11}, a_{12}, a_{13}, \ldots a_{1(n-1)}$, for each we have two options '1' or '–1', but for a_{1n} we have only one way depending upon the product $(a_{11} \times a_{12} \times a_{13} \times \ldots \times a_{1(n-1)})$.

Therefore, for R_1 we have 2^{n-1} ways.

Similarly, for $R_2, R_3, R_4, \ldots, R_{n-1}$ we have 2^{n-1} ways.

For R_n we have only one way.

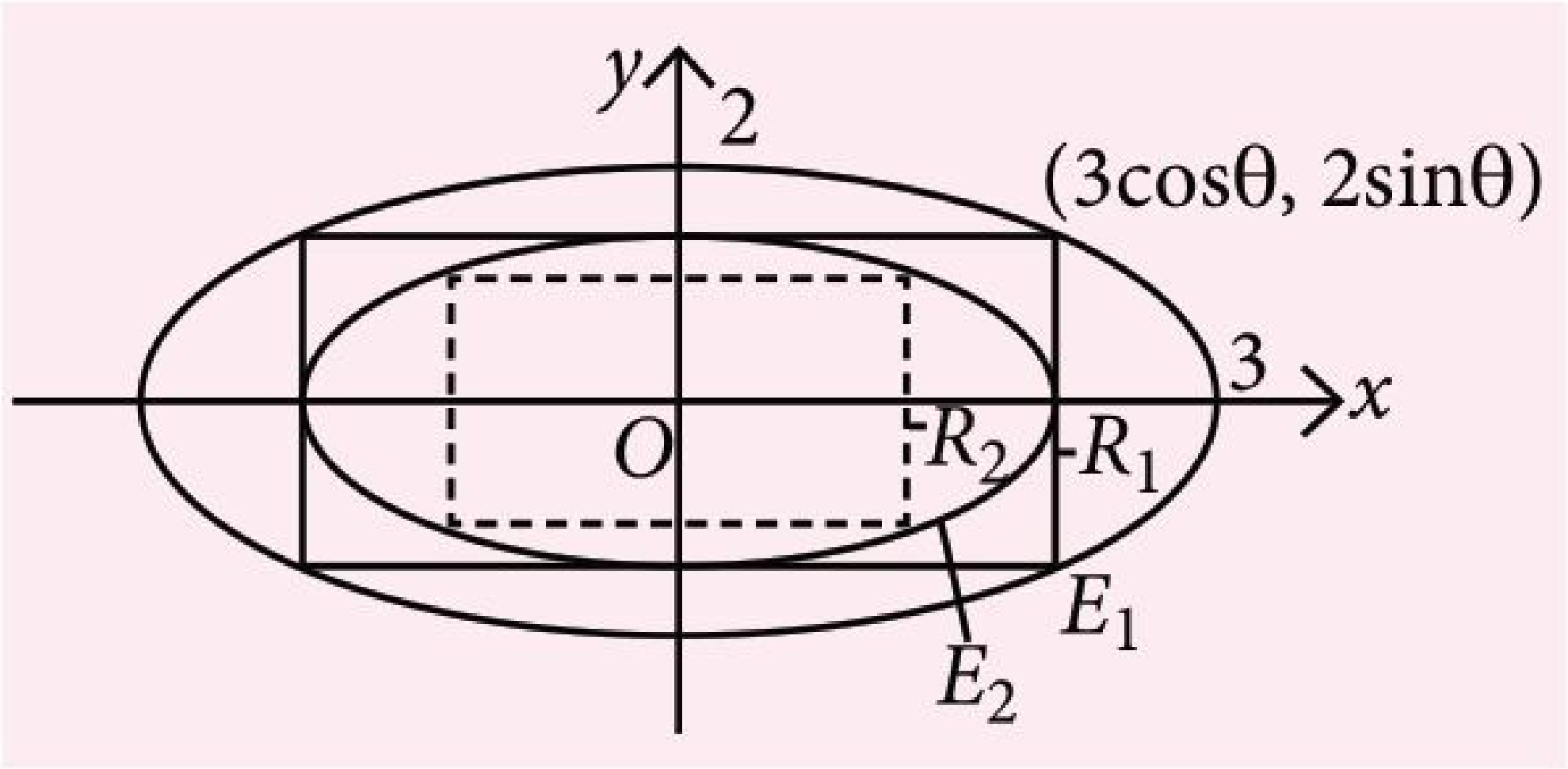
Hence total number of ways $= (2^{n-1})^{n-1} = 2^{(n-1)^2}$

For S_3, we have $2^{(3-1)^2} = 16$ elements.

11. (b, c) : The area of rectangle,

$R_1 = (6\cos\theta)(4\sin\theta) = 12\sin2\theta = 12$ [$\because$ Max$(\sin2\theta) = 1$]

Thus, $\theta = \pi/4$ and, the point is $\left(\frac{3}{\sqrt{2}},\ \frac{2}{\sqrt{2}}\right)$.



Now, $a_2 = \frac{3}{\sqrt{2}},\ b_2 = \frac{2}{\sqrt{2}}$

∴ Area of $R_2 = \left(\frac{6}{\sqrt{2}}\cos\theta\right)\left(\frac{4}{\sqrt{2}}\sin\theta\right) = \frac{12}{2}\sin 2\theta = \frac{12}{2}$

Now, as the ellipse E_n have the largest area inscribed in the rectangle R_{n-1}.

∴ $a_n = \frac{3}{(\sqrt{2})^{n-1}},\ b_n = \frac{2}{(\sqrt{2})^{n-1}}$ and area of $R_n = \frac{12}{2^{n-1}}$

The distance of focus from the centre in $E_9 = a_9e_9$

$$= \sqrt{a_9^2 - b_9^2} = \sqrt{\left(\frac{3}{2^4}\right)^2 - \left(\frac{2}{2^4}\right)^2} = \frac{\sqrt{5}}{16}$$

All ellipse have same eccentricity.

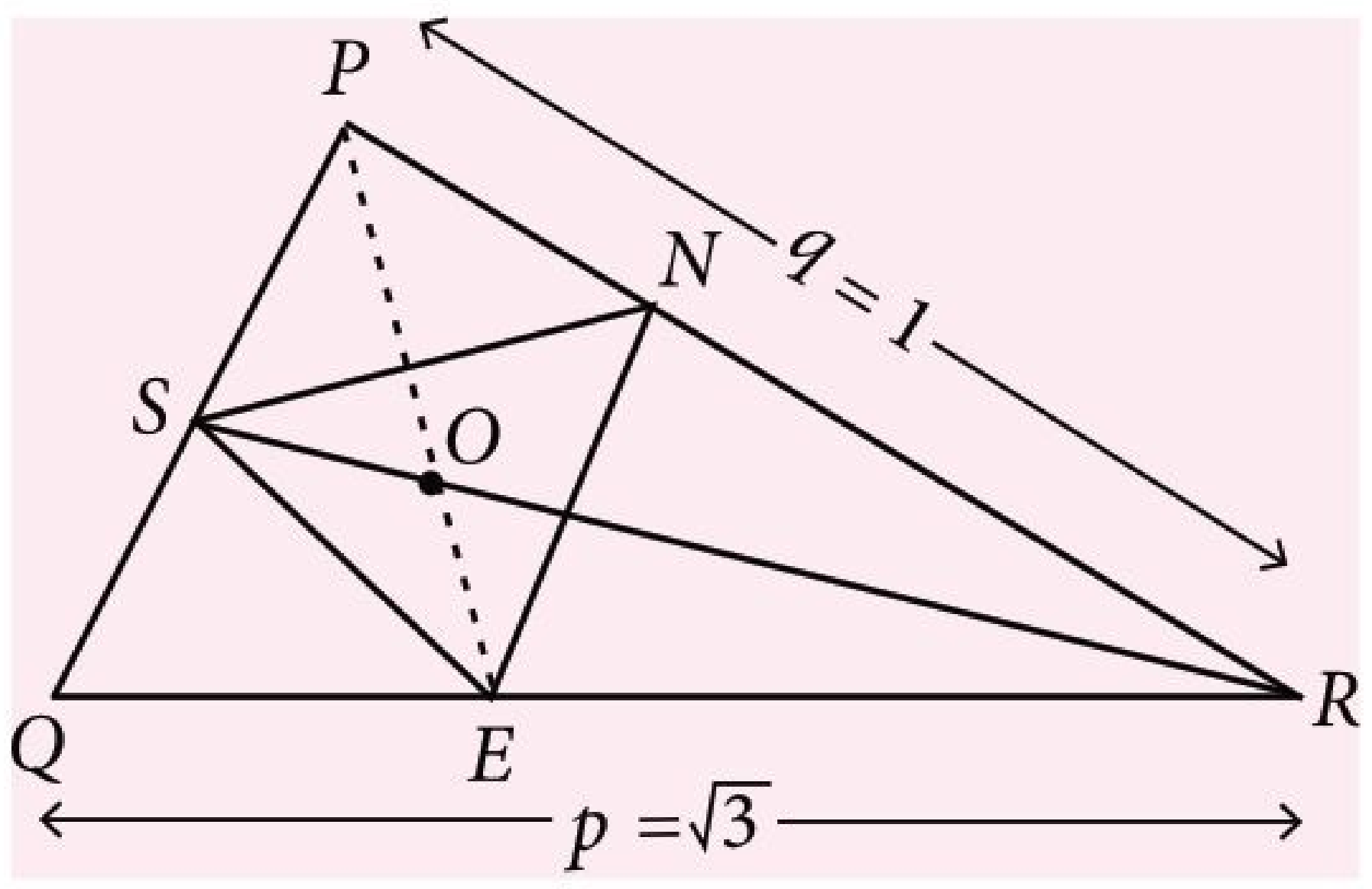
Also, latus rectum of $E_9 = \frac{2b_9^2}{a_9} = \frac{2\cdot\left(\frac{2}{2^4}\right)^2}{\left(\frac{3}{2^4}\right)} = \frac{2}{2^6}\cdot\frac{2^4}{3} = \frac{1}{6}$

$$\sum_{n=1}^{\infty}(\text{area of } R_n) = 12 + \frac{12}{2} + \frac{12}{4} + \ldots\ldots\ \infty = 24$$

∴ $\sum_{n=1}^{N}(\text{area of } R_n) < 24$, for each positive integer N.

12. (a, c, d) : Using sine rule, we have

$$\frac{\sin P}{\sqrt{3}} = \frac{\sin Q}{1} = \frac{1}{2(1)} = \frac{1}{2} \Rightarrow \sin P = \frac{\sqrt{3}}{2}, \sin Q = \frac{1}{2}$$



Thus $P = \pi/3$ or $2\pi/3$ and $Q = \pi/6$ or $5\pi/6$

As $p > q$, so we have $P > Q$.

∴ $P = \frac{2\pi}{3}$ and $Q = R = \frac{\pi}{6}$.

The other option $P = \pi/3$, $Q = \pi/6$ leads to $R = \pi/2$, which is not possible, as given triangle is not right-angled triangle.

⇒ ΔPQR is an isosceles triangle and so PE will be the median.

⇒ O will be the centroid.

∴ $\frac{r}{\sin R} = 2 \Rightarrow r = 2\sin\frac{\pi}{6} = 2\times\frac{1}{2} = 1.$

Now, $[SEN] = \frac{1}{4}[PQR]$, where $[PQR]$ denotes the area of the triangle PQR.

$[SOE] = \frac{1}{3}[SEN] = \frac{1}{12}[PQR]$

$= \frac{1}{12}\left(\frac{1}{2}PR\cdot QR\cdot \sin R\right) = \frac{1}{12}\cdot\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{48}$

Radius of incircle of $\Delta PQR = \Delta/s$

$$= \frac{\sqrt{3}/4}{\frac{\sqrt{3}+2}{2}} = \frac{\sqrt{3}}{2(\sqrt{3}+2)} = \frac{\sqrt{3}}{2}(2-\sqrt{3})$$

Again, $RS = \frac{1}{2}\sqrt{2\cdot 3 + 2\cdot 1 - 1} = \frac{\sqrt{7}}{2}$

and $OE = \frac{1}{3}PE = \frac{1}{3}\cdot\frac{1}{2}\sqrt{2\cdot 1^2 + 2\cdot 1^2 - 3} = \frac{1}{6}$

13. (a, c, d) : We have,

$$f(x) = \begin{cases} (x+1)^5 - 2x, & x < 0; \\ x^2 - x + 1, & 0 \le x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \ge 3. \end{cases}$$

$$\text{Now, } f'(x) = \begin{cases} 5(x+1)^4 - 2, & x < 0 \\ 2x - 1, & 0 < x < 1 \\ 2x^2 - 8x + 7, & 1 < x < 3 \\ \log_e(x-2), & x > 3 \end{cases}$$

$$\text{and } f''(x) = \begin{cases} 2, & 0 < x < 1 \\ 4x - 8, & 1 < x < 3 \end{cases}$$

∵ $\lim_{x\to 1^-} f''(x) \ne \lim_{x\to 1^+} f''(x)$

and $f''(x)$ changes sign from positive to negative.

∴ $f'(x)$ has a local maxima at $x = 1$ and $f'(x)$ is not differentiable at $x = 1$.

∵ $f'(-1) < 0$. ∴ f is not increasing on $(-\infty, 0)$

For $x < 0$, $\lim_{x\to -\infty} f(x) = -\infty$, $\lim_{x\to 0^-} f(x) = 1$

∴ The range of f in $(-\infty, 0)$ contains $(-\infty, 1)$.

For $3 < x < \infty$, $f'(x) > 0$. ∴ f is increasing and $f(3) = \frac{1}{3}$.

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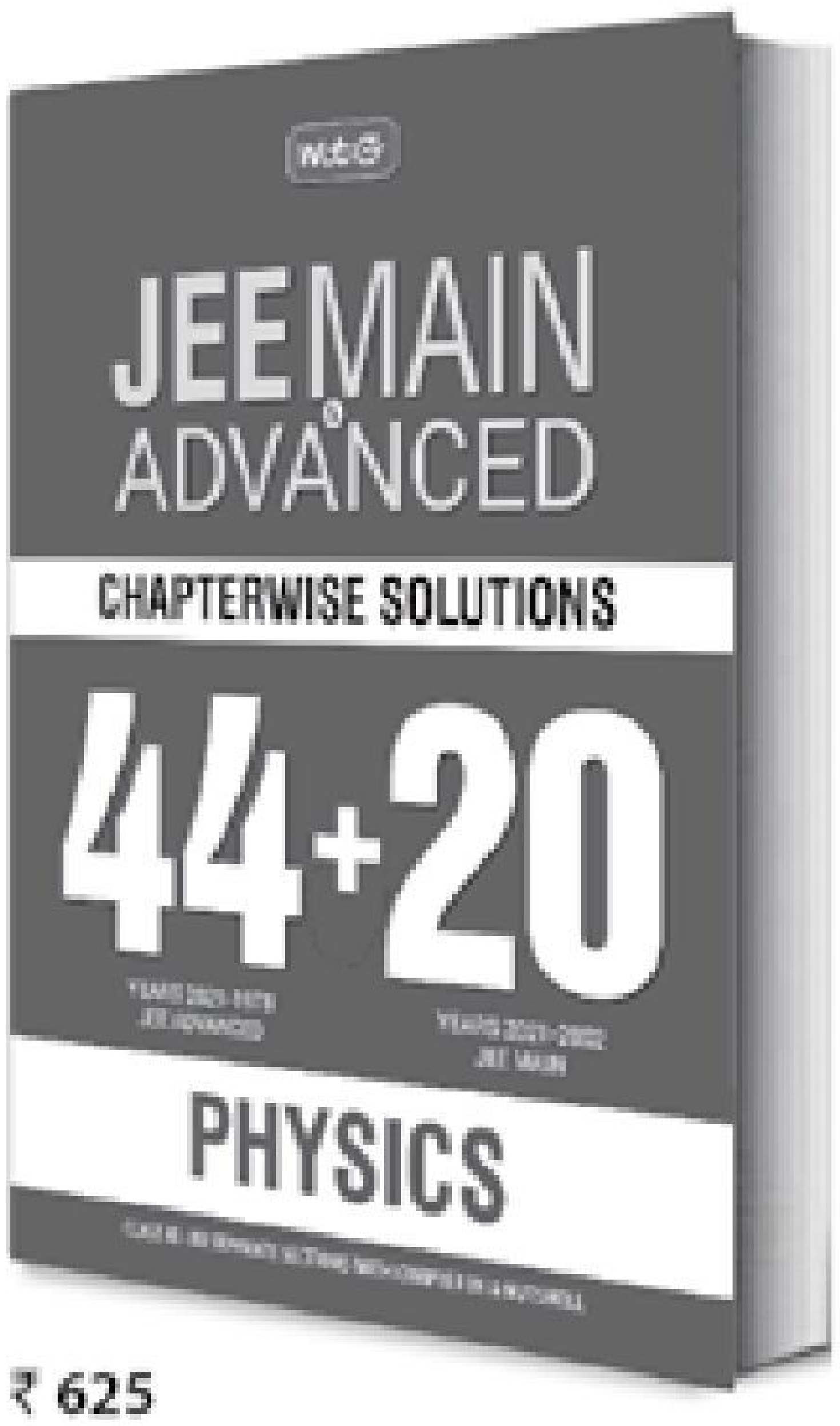
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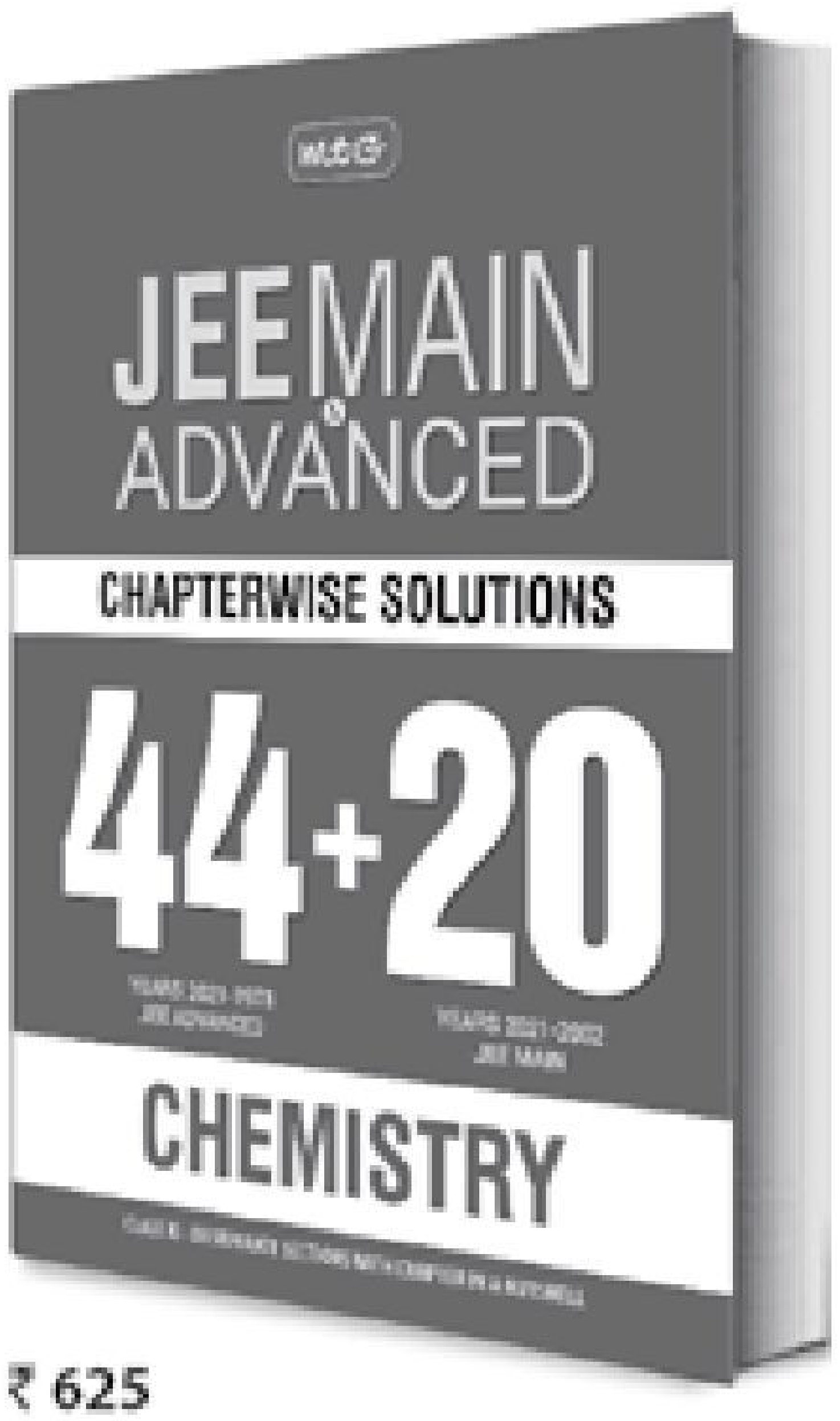
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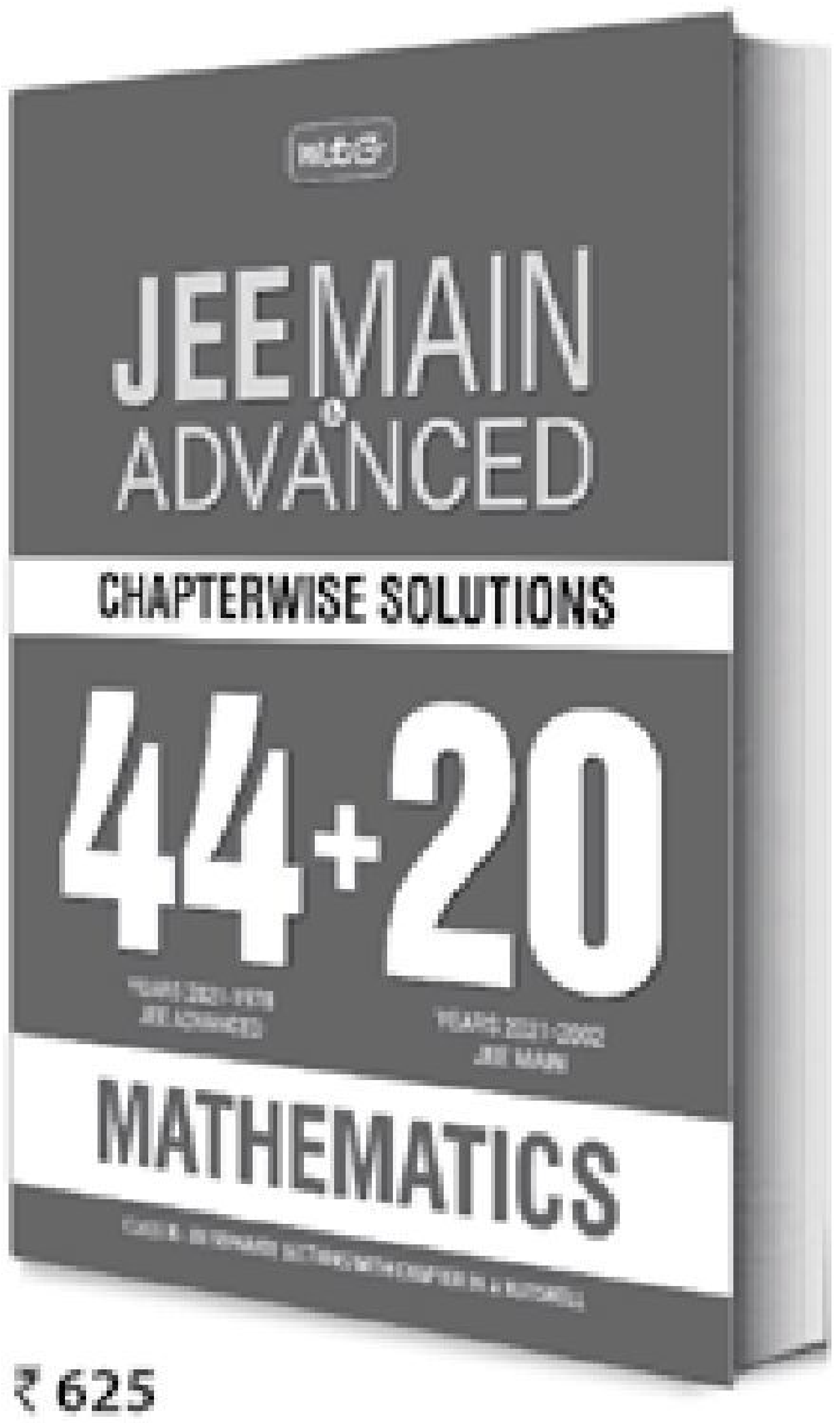
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So, the range of f contains $\left(\frac{1}{3}, \infty\right)$.

Hence, the range of f is whole of R.

14. (a, b, c): Since α and β are roots of $x^2 - x - 1 = 0$.

$\therefore \quad \alpha + \beta = 1, \alpha\beta = -1$ ...(i)

$$\sum_{n=1}^{\infty}\frac{a_n}{10^n} = \frac{1}{\alpha-\beta}\cdot\sum_{n=1}^{\infty}\left\{\left(\frac{\alpha}{10}\right)^n - \left(\frac{\beta}{10}\right)^n\right\}$$

$$= \frac{1}{\alpha-\beta}\left\{\frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} - \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}\right\} = \frac{1}{\alpha-\beta}\left[\frac{\frac{\alpha}{10}-\frac{\beta}{10}}{1-\left(\frac{\alpha+\beta}{10}\right)+\frac{\alpha\beta}{100}}\right]$$

$$= \frac{1}{\alpha-\beta}\left[\frac{10(\alpha-\beta)}{100-10(\alpha+\beta)+\alpha\beta}\right] = \frac{10}{89}$$

Again, $b_n = a_{n+1} + a_{n-1}, n \ge 2$

$$= \frac{(\alpha^{n+1}-\beta^{n+1})+(\alpha^{n-1}-\beta^{n-1})}{\alpha-\beta}$$

$$= \frac{(\alpha^{n+1}-\beta^{n+1})+(-\alpha^{n}\beta+\alpha\beta^{n})}{\alpha-\beta}$$

$(\because \alpha\beta^n = \alpha\beta \cdot \beta^{n-1} = -\beta^{n-1})$

$$= \frac{(\alpha-\beta)(\alpha^n+\beta^n)}{\alpha-\beta} = \alpha^n + \beta^n$$

Also, for $n = 1$, $b_1 = 1 = \alpha + \beta$

$\therefore \quad b_n = \alpha^n + \beta^n, n \ge 1$

$$\sum_{n=1}^{\infty}\frac{b_n}{10^n} = \sum_{n=1}^{\infty}\frac{(\alpha^n+\beta^n)}{10^n} = \sum_{n=1}^{\infty}\left\{\left(\frac{\alpha}{10}\right)^n + \left(\frac{\beta}{10}\right)^n\right\}$$

$$= \frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}} = \frac{\frac{\alpha}{10}+\frac{\beta}{10}-\frac{2\alpha\beta}{100}}{\frac{100-10(\alpha+\beta)+\alpha\beta}{100}}$$

$$= \frac{10(\alpha+\beta)-2\alpha\beta}{100-10(\alpha+\beta)+\alpha\beta} = \frac{12}{89}$$

Clearly, $a_{k+2} - a_{k+1} - a_k = 0$

$$\left[\because a_{k+2} = \frac{\alpha^{k+2}-\beta^{k+2}}{\alpha-\beta} = \frac{\alpha^k(\alpha+1)-\beta^k(\beta+1)}{\alpha-\beta} = a_{k+1} + a_k\right]$$

$\Rightarrow \quad a_k = a_{k+2} - a_{k+1}$

Now, $a_1 = a_3 - a_2, a_2 = a_4 - a_3$

....

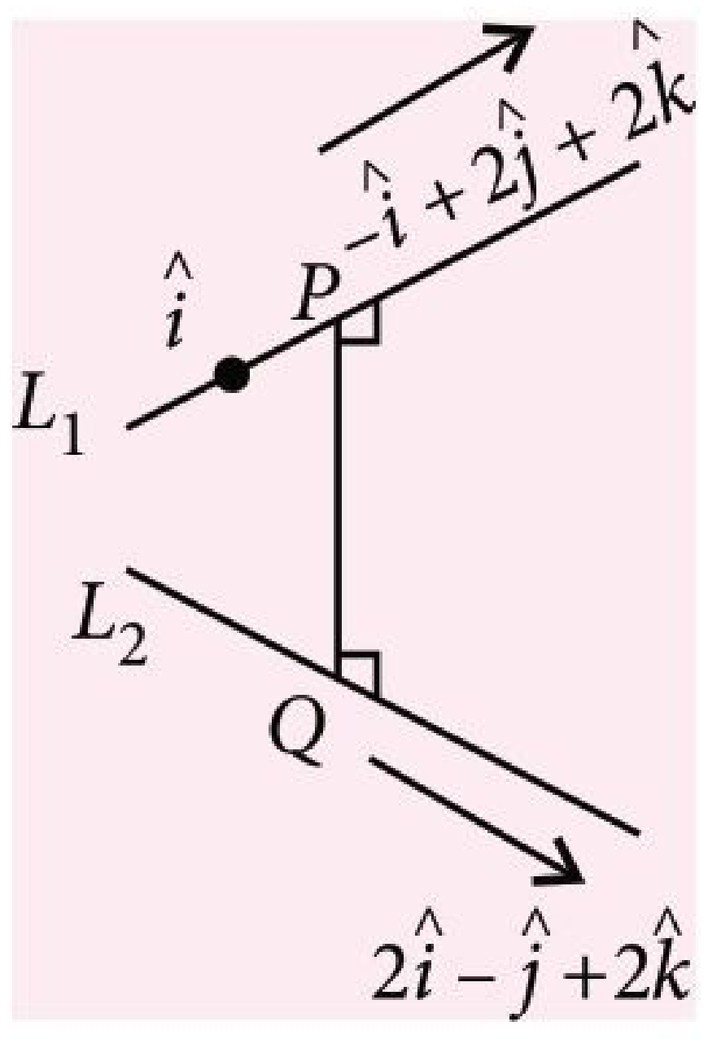
....

$a_n = a_{n+2} - a_{n+1}$

Adding all above the equations, we get

$a_1 + a_2 + \ldots + a_n = a_{n+2} - a_2$

$$= a_{n+2} - \frac{\alpha^2-\beta^2}{\alpha-\beta} = a_{n+2} - (\alpha+\beta) = a_{n+2} - 1 \text{[Using (i)]}$$



15. (a, b, c) : Let P and Q be taken on lines L_1 and L_2 respectively. Then, $P(1 - \lambda, 2\lambda, 2\lambda)$ and $Q(2\mu, -\mu, 2\mu)$

Also,

$$\overrightarrow{PQ} = (2\mu+\lambda-1, -\mu-2\lambda, 2\mu-2\lambda) \quad \text{... (i)}$$

Now, vector along the common perpendicular is

$$(-\hat{i}+2\hat{j}+2\hat{k}) \times (2\hat{i}-\hat{j}+2\hat{k}) = (2\hat{i}+2\hat{j}-\hat{k}) \quad \text{... (ii)}$$

Clearly (i) will be parallel to (ii) and so we get

$$\frac{2\mu+\lambda-1}{2} = \frac{-\mu-2\lambda}{2} = \frac{2\mu-2\lambda}{-1}$$

We have $2\mu + \lambda - 1 = -\mu - 2\lambda$ *i.e.*, $3\mu + 3\lambda = 1$ ... (iii)

Again, $-2\mu - \lambda + 1 = 4\mu - 4\lambda \Rightarrow 6\mu - 3\lambda = 1$ ... (iv)

Solving (iii) and (iv), we have $\mu = 2/9, \lambda = 1/9$

So P is $\left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right)$ and Q is $\left(\frac{4}{9}, -\frac{2}{9}, \frac{4}{9}\right)$

and $\left(\frac{2}{3}, 0, \frac{1}{3}\right)$ lies on PQ, at the midpoint of PQ.

16. (a, c) : The bag B_1 has 5 red and 5 green balls.

Bag B_2 has 3 red and 5 green balls

Bag B_3 has 5 red and 3 green balls

Clearly, $P\left(\frac{G}{B_3}\right) = \frac{3}{8}$

$$P(G) = P(B_1)\cdot P\left(\frac{G}{B_1}\right) + P(B_2)\cdot P\left(\frac{G}{B_2}\right) + P(B_3)\cdot P\left(\frac{G}{B_3}\right)$$

$$= \frac{3}{10}\cdot\frac{1}{2} + \frac{3}{10}\cdot\frac{5}{8} + \frac{4}{10}\cdot\frac{3}{8} = \frac{39}{80}$$

Now, $P(G \cap B_3) = P\left(\frac{G}{B_3}\right)P(B_3) = \frac{3}{8}\times\frac{4}{10} = \frac{3}{20}$

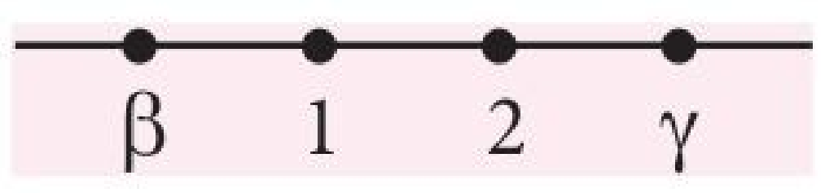
and $P\left(\frac{B_3}{G}\right) = \frac{P(G \cap B_3)}{P(G)} = \frac{3/20}{39/80} = \frac{4}{13}$

17. (3)

18. (390) : Let $f(x) = (\alpha - 5)x^2 - 2\alpha x + \alpha - 4 \ (\alpha \ne 5)$

Let roots of $f(x) = 0$ are β, γ respectively.

As 1 and 2 lies between the roots of $f(x) = 0$, we get $D > 0$, $(\alpha - 5) f(1) < 0$ and $(\alpha - 5) f(2) < 0$



where $f(1) = (\alpha - 5) - 2\alpha + \alpha - 4 = -9$

and $f(2) = 4(\alpha - 5) - 4\alpha + \alpha - 4 = \alpha - 24$

$D > 0 \Rightarrow 4\alpha^2 - 4(\alpha - 5)(\alpha - 4) > 0$

$$\Rightarrow \quad 9\alpha - 20 > 0 \Rightarrow \alpha \in \left(\frac{20}{9}, \infty\right) \quad \text{...(i)}$$

$(\alpha - 5) f(1) < 0 \Rightarrow (\alpha - 5)(-9) < 0$

$\Rightarrow \alpha - 5 > 0 \Rightarrow \alpha \in (5, \infty)$ ...(ii)

$(\alpha - 5) f(2) < 0 \Rightarrow (\alpha - 5)(\alpha - 24) < 0$

$\Rightarrow \alpha \in (5, 24)$ ...(iii)

From (i), (ii) and (iii),
we get $\alpha \in (5, 24)$

$\Rightarrow \alpha = 6, 7, 8, 9, 10, 11, 12, \ldots, 23$

$\therefore$ Sum of all integral values of $\alpha = 6 + 7 + \ldots + 23$, which is an A.P. having $a = 6$, $l = 23$, $n = 18$

Let S = sum of all integral values of α

$$= \frac{n}{2}(a+l) = \frac{18}{2}(6+23) = 9 \times 29 = 3^2 \times 29^1$$

$\therefore$ Sum of all divisors of $S = \dfrac{3^3-1}{3-1} \times \dfrac{29^2-1}{29-1}$

19. (30)

PAPER-2

1. (a, b, c, d) : Given, trajectory is

$$\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1 \qquad \ldots\text{(i)}$$

$$\Rightarrow \frac{y}{a^2+\lambda} = \frac{1}{y}\left(\frac{a^2-x^2}{a^2}\right) \qquad \ldots\text{(ii)}$$

Differentiating (i) w.r.t. x we get

$$\frac{2x}{a^2} + \frac{2y}{a^2+\lambda}\frac{dy}{dx} = 0 \text{ or } \frac{x}{a^2} + \frac{y}{a^2+\lambda}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{x}{a^2} + \frac{a^2-x^2}{a^2 y}\frac{dy}{dx} = 0 \qquad \text{(using (ii))} \qquad \ldots\text{(iii)}$$

$\therefore$ Differential equation of orthogonal trajectory is

$$\frac{x}{a^2} - \frac{a^2-x^2}{a^2 y}\frac{dx}{dy} = 0 \left(\text{Replacing } \frac{dy}{dx} \text{ by } -\frac{dx}{dy} \text{ in (iii)}\right)$$

$$\Rightarrow ydy + \frac{x^2-a^2}{x}dx = 0 \Rightarrow ydy + \left(x - \frac{a^2}{x}\right)dx = 0$$

$$\Rightarrow \frac{y^2}{2} + \frac{x^2}{2} - a^2 \log x = c, \text{ using (1, 1) we get } c = 1$$

$$\Rightarrow \frac{y^2}{2} + \frac{x^2}{2} - (a^2 \log x + 1) = 0$$

$\Rightarrow x^2 + y^2 - 2(a^2 \log x + 1) = 0$

$\Rightarrow x^{k_1} + y^{k_2} - k_3(a^{k_4} \log x + 1) = 0$

$\Rightarrow k_1 = k_2 = k_3 = k_4 = 2$ and $k_1^5 = 32 = 16(2) = 16k_2$

2. (b, c, d) : (a) $\vec{a}\times(\vec{b}\times\vec{c}) + \vec{b}\times(\vec{c}\times\vec{a}) + \vec{c}\times(\vec{a}\times\vec{b}) = \vec{0}$

$\Rightarrow$ vectors are coplanar, so do not form RHS.

(b) $(\vec{a}\times\vec{b})\times\vec{c}$, $\vec{a}\times\vec{b}$, $\vec{c}$ in this order form RHS.

$\Rightarrow$ $\vec{c}$, $(\vec{a}\times\vec{b})\times\vec{c}$, $\vec{a}\times\vec{b}$ also form RHS as they are in same cyclic order.

(c) $\vec{a}+\vec{b}+\vec{c} = \vec{0} \Rightarrow (\vec{a}+\vec{b}+\vec{c})^2 = 0$

$\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 = -2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

Hence, $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} < 0$

(d) $\vec{a}+\vec{b}+\vec{c} = \vec{0} \Rightarrow \vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

Using this we get the result.

3 (a, b, c, d) : Conditions for the plane $ax + by + cz + d = 0$ to contain the line $\dfrac{x-\alpha}{l} = \dfrac{y-\beta}{m} = \dfrac{z-\gamma}{n}$ are $a\alpha + b\beta + c\gamma + d = 0$ and $al + bm + cn = 0$.

Here the condition is satisfied for option (a).

For option (c), normal to plane and given line have same d.r.'s.

$\therefore$ The line is perpendicular to the given plane.

For option (b), clearly the point satisfies the given plane.

The d.r.'s of normal to the plane in option (d) are (1, –2, 7) which are same as the d.r.'s of normal to the given plane.

$\therefore$ The plane given in option (d) is parallel to given plane.

4. (c, d) : $f'(x) = \dfrac{192x^3}{2+\sin^4 \pi x}$, $x \in \left(\dfrac{1}{2}, 1\right)$

$$\min. f'(x) = \frac{192x^3}{2+1} = 64x^3$$

$$\text{and max. } f'(x) = \frac{192x^3}{2+0} = 96x^3$$

$\therefore$ $64x^3 \le f'(x) \le 96x^3$

$$\Rightarrow \int_{1/2}^{x} 64x^3 dx \le \int_{1/2}^{x} f'(x)dx \le \int_{1/2}^{x} 96x^3 dx$$

$$\Rightarrow \left[\frac{64x^4}{4}\right]_{1/2}^{x} \le [f(x)]_{1/2}^{x} \le \left[\frac{96x^4}{4}\right]_{1/2}^{x}$$

$$\Rightarrow 16\left(x^4 - \frac{1}{16}\right) \le f(x) - f\left(\frac{1}{2}\right) \le 24\left(x^4 - \frac{1}{16}\right)$$

$$\Rightarrow 16x^4 - 1 \le f(x) \le 24x^4 - \frac{3}{2} \qquad \left(\because f\left(\frac{1}{2}\right) = 0\right)$$

$$\Rightarrow \int_{1/2}^{1} (16x^4 - 1)dx \le \int_{1/2}^{1} f(x)dx \le \int_{1/2}^{1} \left(24x^4 - \frac{3}{2}\right)dx$$

$$\Rightarrow \left[\frac{16x^5}{5} - x\right]_{1/2}^{1} \le \int_{1/2}^{1} f(x)dx \le \left[\frac{24x^5}{5} - \frac{3}{2}x\right]_{1/2}^{1}$$

$$\Rightarrow \left(\frac{16}{5}-1\right)-\left(\frac{16}{5}\times\frac{1}{32}-\frac{1}{2}\right)\le\int_{1/2}^{1} f(x)dx\le\left(\frac{24}{5}-\frac{3}{2}\right)$$
$$-\left(\frac{24}{5}\times\frac{1}{32}-\frac{3}{2}\times\frac{1}{2}\right)$$

$$\Rightarrow 2.6\le\int_{1/2}^{1} f(x)dx\le 3.9 \quad \because \quad m\le\int_{1/2}^{1} f(x)dx\le M$$

5. (a, b, c, d) : $\left|\int_0^r f(x)dx\right|=\frac{2}{5}\Rightarrow\int_0^r f(x)dx=\frac{-2}{5}$

[$\because$ Bounded area lies below the x-axis]

$\int_r^s f(x)dx=2$

$\left|\int_s^6 f(x)dx\right|=12\Rightarrow\int_s^6 f(x)dx=-12$

[$\because$ Bounded area lies below the x-axis]

Now, $m=\int_s^0 f(x)dx=-\int_0^s f(x)dx$

$=-\left[\int_0^r f(x)dx+\int_r^s f(x)dx\right]=-\left[\frac{-2}{5}+2\right]=-1.6$

$p=\int_r^6 f(x)dx=\int_r^s f(x)dx+\int_s^6 f(x)dx$

$= 2 - 12 = -10$

$q=\left|\int_0^6 f(x)dx\right|=\left|\int_0^r f(x)dx+\int_r^s f(x)dx+\int_s^6 f(x)dx\right|$

$=\left|\frac{-2}{5}+2-12\right|=10.4$

$n=\int_0^6|f(x)|dx=\int_0^r|f(x)|dx+\int_r^s|f(x)|dx+\int_s^6|f(x)|dx$

$=\frac{2}{5}+2+12=14.4$

$\therefore$ $m = -1.6$; $p = -10$; $q = 10.4$; $n = 14.4$.

6. (c, d) : The range of $f(x)$ is $\left[-1,\frac{\pi}{2}-1\right]$ which is a proper subset of $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Hence $\tan f(x)$ is continuous.

$\frac{1}{f(x)}=\frac{2}{x-2}$, not continuous at $x = 2$

$f^{-1}(x) = 2(x + 1)$, $\frac{1}{f^{-1}(x)}=\frac{1}{2(x+1)}$ is continuous in $[0, \pi]$.

7. (560) : Number of shortest routes through P

= (number of shortest routes from A to P)

× (number of shortest routes from P to C)

$=\frac{5!}{3!\cdot 2!}\times\frac{8!}{3!\cdot 5!}=560.$

8. (350) : Number of shortest routes $=\frac{5!}{3!\cdot 2!}\times 1\times\frac{7!}{3!\cdot 4!}$

$= 350.$

9. (0.125) : $\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$

$=\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{(1-\cos x)^2}\left(\frac{1-\cos x}{x^2}\right)^2=\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$

10. (0.5) : $\lim_{x\to 0}\frac{(1-\cos^2(\sin x))(1-\cos(\sin^2 x))}{x^6}$

$=\lim_{x\to 0}(1+\cos(\sin x))\left(\frac{1-\cos(\sin x)}{\sin^2 x}\right)\frac{\sin^2 x}{x^2}$

$\times\frac{1-\cos(\sin^2 x)}{\sin^4 x}\times\frac{\sin^4 x}{x^4}=2\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{2}$

11. (26) : We know that Q.D. $=\frac{Q_3-Q_1}{2}$

Here, $n = 104 \Rightarrow \frac{n}{4}=\frac{104}{4}=26.00$

and $\frac{3n}{4}=3\times 26.00=78.00$

Hence, Q_1 lies between 26th and 27th items and Q_3 lies between 78th and 79th item.

$\Rightarrow Q_1=\frac{x_{26}+x_{27}}{2}=\frac{26+27}{2}=26.5$

and $Q_3=\frac{x_{78}+x_{79}}{2}=\frac{78+79}{2}=78.5$

$\Rightarrow$ Q.D. $=\frac{Q_3-Q_1}{2}=\frac{78.5-26.5}{2}=\frac{52}{2}=26.$

12. (26) : Mean deviation from median

$=\frac{1}{104}\{|x_1-52.5|+|x_2-52.5|+\ldots+|x_{104}-52.5|\}$

$=\frac{1}{104}\{|1-52.5|+|2-52.5|+\ldots$
$+|52-52.5|+|53-52.5|+\ldots+|104-52.5|\}$

$=\frac{1}{104}\{51.5+50.5+\ldots+0.5+0.5+\ldots+51.5\}$

$=\frac{2}{104}(0.5+1.5+2.5+\ldots+51.5)$

$=\frac{2}{104}\times\frac{52}{2}(0.5+51.5)=\frac{1}{2}(52)=26.$

13. (b) : Given sets $A = \{1, 2, 3\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{3, 4, 5, 6, 7, 8, 9\}$

Total number of ways (one number from each set A, B, C)

$= {}^3C_1 \times {}^5C_1 \times {}^7C_1 = 105$

Favourable cases : **Case (i) :** All three numbers be odd one from each set A, B, C

$\therefore$ Required ways $= {}^2C_1 \times {}^3C_1 \times {}^4C_1 = 24$

Case (ii) : One odd number from set A & one even number from set B & C each.

$\therefore$ Required ways $= {}^2C_1 \times {}^2C_1 \times {}^3C_1 = 12$

Case (iii) : One odd number from set B & one even number from set A & C each.

$\therefore$ Required ways $= {}^3C_1 \times {}^1C_1 \times {}^3C_1 = 9$

Case (iv) : One odd number from set C & one even number from set A & B each.

$\therefore$ Required ways $= {}^4C_1 \times {}^1C_1 \times {}^2C_1 = 8$

Favourable cases = 24 + 12 + 9 + 8 = 53

$\therefore$ Required probability $= \dfrac{53}{105}$

14. (b) : As $2x_2 = x_1 + x_3$

Number of ways = 10

$\therefore$ Required probability $= \dfrac{10}{105}$

15. (b) : The number of students passed in English and Mathematics but not in Science

$= n(E \cap M) - n(E \cap M \cap S) = 6 - 4 = 2$

16. (c) : The number of students passed in Mathematics only $= n(M \cap \overline{E} \cap \overline{S}) = n(M) - n(M \cap E) - n(M \cap S) + n(M \cap E \cap S) = 12 - 6 - 7 + 4 = 16 - 13 = 3$

17. (3) : Given, $y = \left(1+\dfrac{1}{x}\right)^x$

Taking logarithm on both sides, we get

$$\log y = x\log\left(1+\frac{1}{x}\right)$$

Differentiating w.r.t. x, we get

$$\frac{1}{y}y_1(x) = \frac{x^2}{x+1}\left(-\frac{1}{x^2}\right)+\log\left(1+\frac{1}{x}\right) = -\frac{1}{x+1}+\log\left(1+\frac{1}{x}\right)$$

Since, $y(2) = (1 + 1/2)^2 = 9/4$

So, $y_1(2) = \left(\dfrac{9}{4}\right)\left(-\dfrac{1}{3}+\log\dfrac{3}{2}\right)$

Multiplying (i) by y and then differentiating, we get

$$y_2(x) = y_1(x)\left(-\frac{1}{x+1}+\log\left(1+\frac{1}{x}\right)\right)+y(x)\times \left(\frac{1}{(x+1)^2}+\frac{x}{x+1}\left(-\frac{1}{x^2}\right)\right)$$

So, $y_2(2) = y_1(2)\left(-\dfrac{1}{3}+\log\dfrac{3}{2}\right)+y(2)\left(\dfrac{1}{9}-\dfrac{1}{6}\right)$

$$\Rightarrow y_2(2) = \left(\frac{9}{4}\right)\left(-\frac{1}{3}+\log\frac{3}{2}\right)^2-\frac{1}{8}$$

$$\Rightarrow \quad y_2(2)+\frac{1}{8} = \left(\frac{3}{2}\right)^2\left(\log\frac{3}{2}-\frac{1}{3}\right)^2$$

$$\Rightarrow \quad \sqrt{y_2(2)+\frac{1}{8}} = \frac{3}{2}\left(\log\frac{3}{2}-\frac{1}{3}\right) \qquad \therefore \quad \frac{2\sqrt{y_2(2)+\frac{1}{8}}}{\left(\log\frac{3}{2}-\frac{1}{3}\right)} = 3$$

18. (1) : Let $P(x, y)$ be a point on $x^ny = a^n$ …(1)

Then $nx^{n-1}y+x^n\dfrac{dy}{dx} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-ny}{x} = \frac{-na^n}{x^{n+1}} \qquad \text{[By (1)]}$$

$\therefore$ Equation of tangent at P is

$$Y-y = -\frac{na^n}{x^{n+1}}(X-x)$$

It meets x-axis at $A\left(\dfrac{yx^{n+1}}{na^n}+x, 0\right)$

and y-axis at $B\left(0, y+\dfrac{na^n}{x^n}\right)$

$\therefore$ Area of $\Delta OAB = \dfrac{1}{2}OA \times OB$

$$= \frac{1}{2}\left(\frac{yx^{n+1}}{na^n}+x\right)\left(y+\frac{na^n}{x^n}\right)$$

$$= \frac{1}{2}\left(\frac{x}{n}+x\right)\left(\frac{a^n}{x^n}+\frac{na^n}{x^n}\right) \qquad \text{[By (1)]}$$

$= \dfrac{\frac{1}{2}(n+1)^2a^n}{nx^{n-1}}$, which is constant,

if $n - 1 = 0$, *i.e.* if $n = 1$.

19. (1) : We have, $4\cos^4 x - 2\cos 2x - \dfrac{1}{2}\cos 4x - x^7$

$= 4\cos^4 x - 2(2\cos^2 x - 1) - \dfrac{1}{2}(2\cos^2 2x - 1) - x^7$

$$= 4\cos^4 x - 4\cos^2 x + 2 - (2\cos^2 x - 1)^2 + \frac{1}{2} - x^7 = \frac{3}{2} - x^7$$

$\therefore \quad g(x) = \left(\dfrac{3}{2}-x^7\right)^{1/7} \Rightarrow g(g(x)) = \left(\dfrac{3}{2}-(g(x))^7\right)^{1/7}$

$= \left(\dfrac{3}{2}-\left(\dfrac{3}{2}-x^7\right)\right)^{1/7} = x$. Hence, $\dfrac{g(g(100))}{100} = \dfrac{100}{100} = 1$

◆ ◆

CUET (UG)

PRACTICE PAPER 2022

Section II of CUET (UG) is Domain specific. In this section of Mathematics 40 questions to be attempted out of 50.

Max. Marks : 200 Time : 45 minutes

Multiple Choice Questions (MCQs)

1. Consider $f : R^+ \to (4, \infty)$ given by $f(x) = x^2 + 4$. If $f(x)$ is invertible, then find the inverse of $f(x)$, where R^+ is the set of all non-negative real numbers.

(a) $\sqrt{y-4}$ (b) $\sqrt{x+4}$

(c) $\sqrt{x-4}$ (d) $\sqrt{y+4}$

2. Find the value of $\int_{-\pi/2}^{\pi/2} |\sin x|\, dx$.

(a) 0 (b) 1 (c) 2 (d) 3

3. $\cos^{-1}[\cos(2\cot^{-1}(\sqrt{2}-1))] =$

(a) $\sqrt{2}-1$ (b) $1+\sqrt{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

4. The differential equation of all 'Simple Harmonic Motions' of given period $\frac{2\pi}{n}$ is

(a) $\frac{d^2x}{dt^2} + nx = 0$ (b) $\frac{d^2x}{dt^2} + n^2x = 0$

(c) $\frac{d^2x}{dt^2} - n^2x = 0$ (d) $\frac{d^2x}{dt^2} + \frac{1}{n^2}x = 0$

5. The identity element for the binary operation $*$ defined on $Q - \{0\}$ as $a * b = \frac{ab}{2}\ \forall\, a, b \in Q - \{0\}$ is

(a) 1 (b) 0

(c) 2 (d) None of these

6. The area bounded by the curve $y = \sec^2 x$, $y = 0$ and $|x| = \frac{\pi}{3}$ is

(a) $\sqrt{3}$ sq. units (b) $\sqrt{2}$ sq. units

(c) $2\sqrt{3}$ sq. units (d) none of these

7. If
$$f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2}, & x<0 \\ a\quad, & x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x>0 \end{cases}$$
is continuous at $x = 0$, then $a =$

(a) 4 (b) 6 (c) 8 (d) 5

8. If the volume of a sphere is increasing at a constant rate, then the rate at which its radius is increasing, is

(a) a constant

(b) proportional to the radius

(c) inversely proportional to the radius

(d) inversely proportional to the surface area

9. The equation of the line joining the points $(-3, 4, 11)$ and $(1, -2, 7)$ is

(a) $\frac{x+3}{2} = \frac{y-4}{3} = \frac{z-11}{4}$

(b) $\frac{x+3}{-2} = \frac{y-4}{3} = \frac{z-11}{2}$

(c) $\frac{x+3}{-2} = \frac{y+4}{3} = \frac{z+11}{4}$

(d) $\frac{x+3}{2} = \frac{y+4}{-3} = \frac{z+11}{2}$

10. If $f : [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals

(a) $\frac{x+\sqrt{x^2-4}}{2}$ (b) $\frac{x}{1+x^2}$

(c) $\frac{x-\sqrt{x^2-4}}{2}$ (d) $1+\sqrt{x^2-4}$

11. Evaluate : $\int \frac{x^3}{x+2}\, dx$

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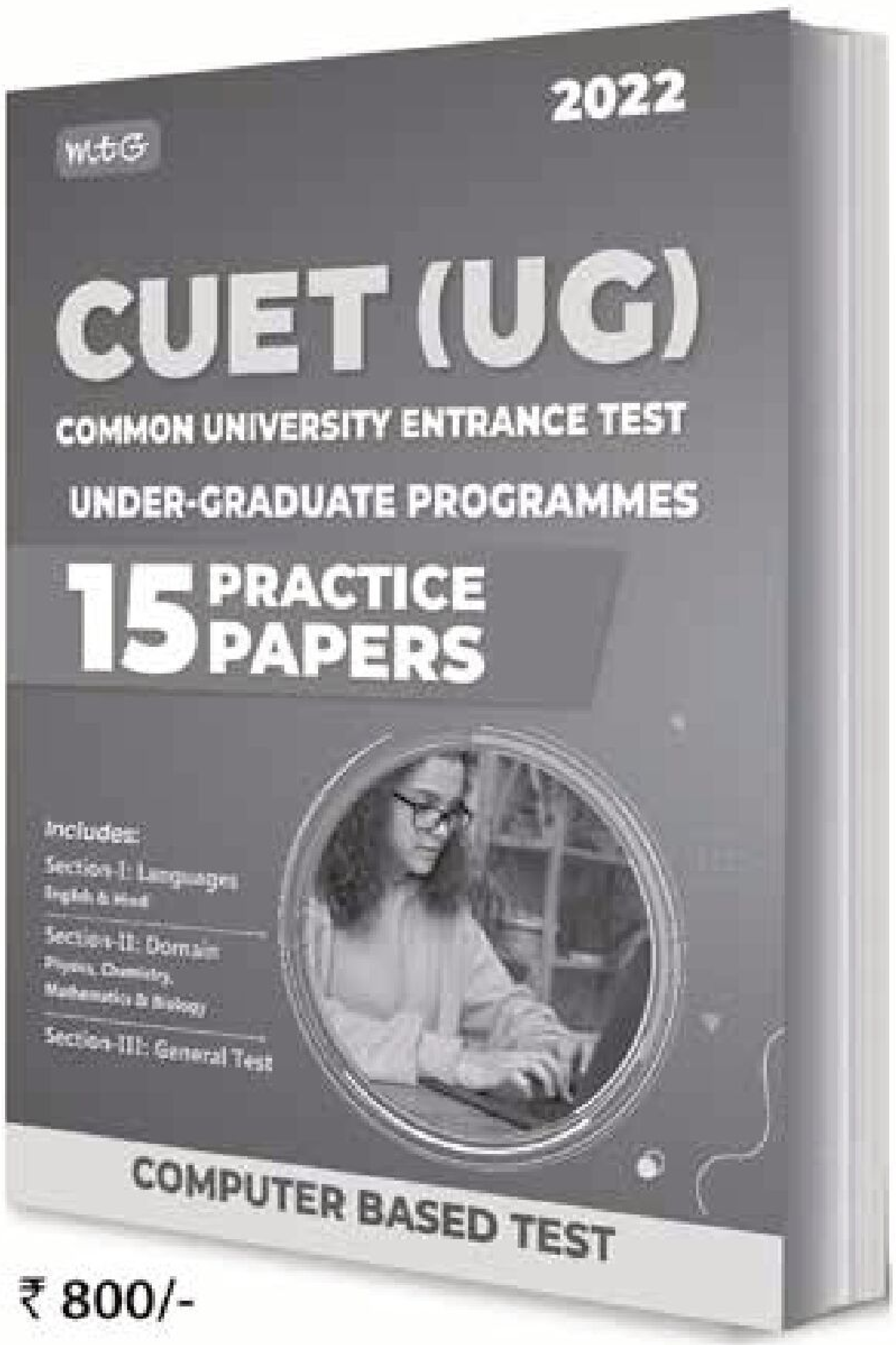
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(a) $\frac{x^3}{3}-x^2-4x-8\log|x+2|+C$

(b) $\frac{x^3}{3}-x^2+4x-8\log|x+2|+C$

(c) $\frac{x^3}{3}+x^2+4x+8\log|x+2|+C$

(d) $\frac{x^3}{3}+x^2+4x-8\log|x+2|+C$

12. If the function $f(x)=\begin{cases}\frac{\sin x^2}{x}; & x\neq 0\\ 0\ ; & x=0\end{cases}$, is differentiable at $x = 0$, then right hand derivative of $f(x)$ at $x = 0$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) –1

13. A new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains 80 grams of proteins and 40 milligrams of iron per kilogram, and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs ₹ 5 per kilogram and rice costs ₹ 4 per kilogram.

(a) ₹ 4.8 (b) ₹ 4.6 (c) ₹ 3.2 (d) ₹ 4

14. Use differentials to find the approximate value of $\sqrt{0.037}$.

(a) 0.1925 (b) 2.053 (c) 1.192 (d) 2.624

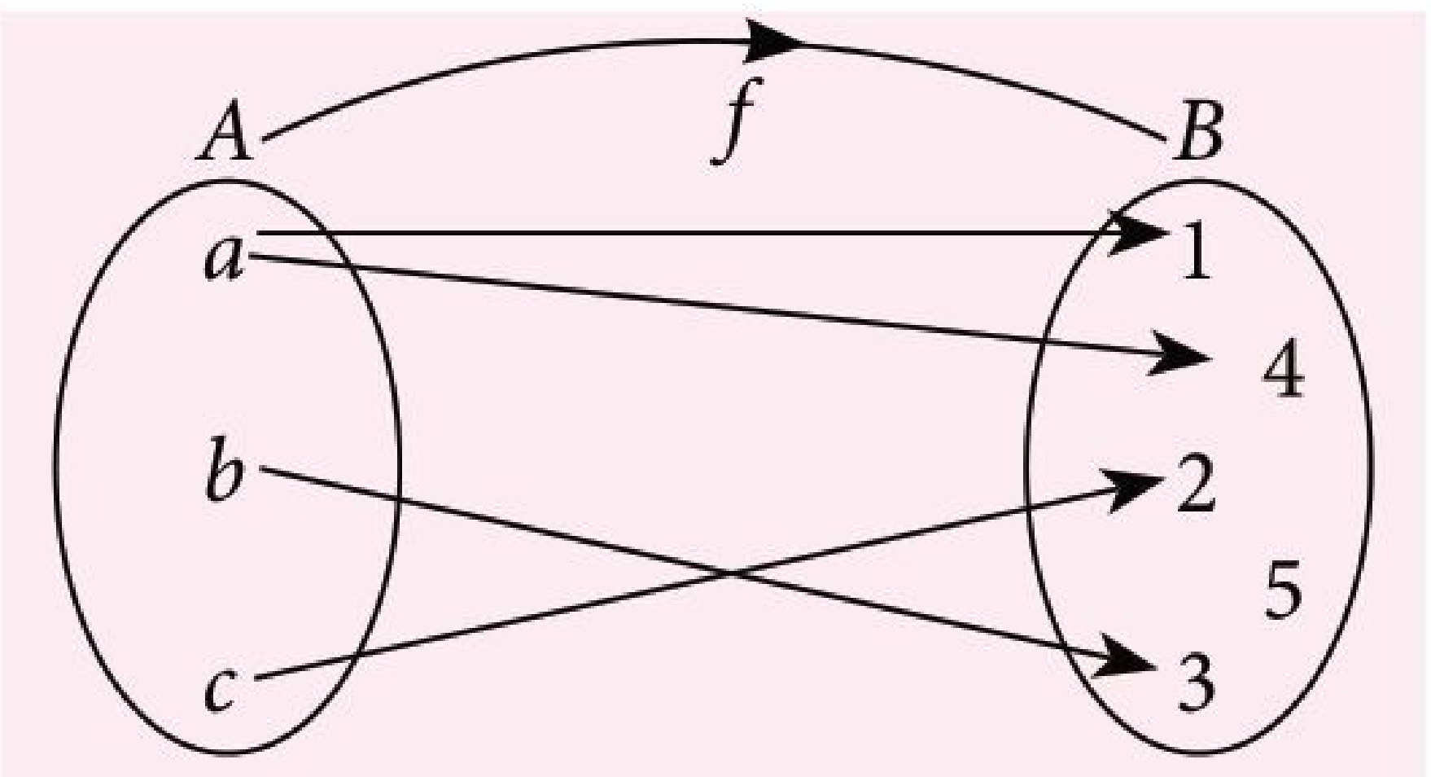
15. A random variable X has the following distribution.

X	1	2	3	4	5	6	7	8
P(X)	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the event $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, $P(E \cup F) =$

(a) 0.87 (b) 0.77 (c) 0.35 (d) 0.50

16. The diagram given below shows that



(a) f is a function from A to B

(b) f is a one-one function from A to B

(c) f is an onto function from A to B

(d) f is not a function.

17. Solution of the differential equation

$$\frac{dx}{dy}-\frac{x\log x}{1+\log x}=\frac{e^y}{1+\log x},$$ if $y(1) = 0$, is

(a) $x^x=e^{ye^y}$ (b) $e^y=x^{e^y}$

(c) $x^x = ye^y$ (d) none of these

18. If the function $f(x)=\begin{cases}\left[\tan\left(\frac{\pi}{4}+x\right)\right]^{1/x} & \text{for } x\neq 0\\ K & \text{for } x=0\end{cases}$

is continuous at $x = 0$, then $K = ?$

(a) e (b) e^{-1} (c) e^2 (d) e^{-2}

19. The vector equation of the line through the points $A(3, 4, -7)$ and $B(1, -1, 6)$ is

(a) $\vec{r}=(3\hat{i}-4\hat{j}-7\hat{k})+\lambda(\hat{i}-\hat{j}+6\hat{k})$

(b) $\vec{r}=(\hat{i}-\hat{j}+6\hat{k})+\lambda(3\hat{i}-4\hat{j}-7\hat{k})$

(c) $\vec{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda(-2\hat{i}-5\hat{j}+13\hat{k})$

(d) $\vec{r}=(\hat{i}-\hat{j}+6\hat{k})+\lambda(4\hat{i}+3\hat{j}-\hat{k})$

20. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, then $x^4 + y^4 + z^4 + 4x^2y^2z^2 =$

(a) $x^2 + y^2 + y^2z^2$ (b) $2(x^2y^2 + y^2z^2 + z^2x^2)$

(c) $(x + y)^2$ (d) $(x + y + z)^2$

21. Maximize $Z = 11x + 8y$, subject to $x \le 4$, $y \le 6$, $x + y \le 6$, $x \ge 0$, $y \ge 0$.

(a) 44 at (4, 2) (b) 60 at (4, 2)

(c) 62 at (4, 0) (d) 48 at (4, 2)

22. The value of c in Mean Value theorem for the function $f(x) = (x - 3)(x - 6)(x - 9)$ in [3, 5] is

(a) $6 \pm \sqrt{13/3}$ (b) $6 + \sqrt{13/3}$

(c) $6 - \sqrt{13/3}$ (d) None of these

23. The area enclosed by the curve $y=\sqrt{x}$ and $x=-\sqrt{y}$, the circle $x^2 + y^2 = 2$ above the x-axis, is

(a) $\frac{\pi}{4}$ sq. units (b) $\frac{3\pi}{2}$ sq. units

(c) π sq. units (d) $\frac{\pi}{2}$ sq. units

24. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining third six in the sixth throw of the die.

(a) $\frac{625}{23329}$ (b) $\frac{621}{25329}$ (c) $\frac{625}{23328}$ (d) $\frac{620}{23328}$

25. The equation of the tangent to the curve $y = (4 - x^2)^{2/3}$ at $x = 2$ is
(a) $x = -2$ (b) $x = 2$
(c) $y = 2$ (d) $y = -1$

26. $[1\ x\ 1]\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$, if $x =$
(a) -7 (b) -11 (c) -2 (d) 14

27. Evaluate : $\int \frac{dx}{\sqrt{1-2x-x^2}}$
(a) $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1+x}{\sqrt{2}}\right)+c$ (b) $\frac{1}{\sqrt{2}}\log(1+x)+c$
(c) $\sin^{-1}\left(\frac{1+x}{\sqrt{2}}\right)+c$ (d) $\frac{1}{\sqrt{2}}\log\left(\frac{1+x}{\sqrt{2}}\right)+c$

28. The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$ is
(a) none (b) one (c) two (d) infinite

29. A brick manufacturer has two depots A and B, with stocks of 30000 and 20000 bricks respectively. He receives orders from three builders P, Q and R for 15000, 20000 and 15000 bricks respectively. The cost (in ₹) of transporting 1000 bricks to the builders from the depots are as given in the table :

To / From	**Transportation cost per 1000 bricks (in ₹)**		
	P	**Q**	**R**
A	40	20	20
B	20	60	40

The manufacturer wishes to find how to fulfil the order so that transportation cost is minimum. Formulate the LPP.
(a) Minimize $Z = 40x - 20y$
Subject to, $x + y \geq 15$, $x + y \leq 30$, $x \geq 15$, $y \leq 20$, $x \geq 0$, $y \geq 0$
(b) Minimize $Z = 40x - 20y$
Subject to, $x + y \geq 15$, $x + y \leq 30$, $x \leq 15$, $y \geq 20$, $x \geq 0$, $y \geq 0$
(c) Minimize $Z = 40x - 20y$
Subject to, $x + y \geq 15$, $x + y \leq 30$, $x \leq 15$, $y \leq 20$, $x \geq 0$, $y \geq 0$
(d) Minimize $Z = 40x - 20y$
Subject to, $x + y \geq 15$, $x + y \leq 30$, $x \geq 15$, $y \geq 20$, $x \geq 0$, $y \geq 0$

30. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = (1+x)(1-x)$, then $f(A)$ is
(a) $-4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $-8\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
(c) $4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (d) None of these

31. A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, the probability that both are dead is
(a) $\frac{33}{56}$ (b) $\frac{9}{64}$ (c) $\frac{1}{14}$ (d) $\frac{3}{28}$

32. The slope of the tangent to the curve
$x = a \sin t$, $y = a\left\{\cos t + \log\left(\tan\frac{t}{2}\right)\right\}$ at the point 't' is

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8	7	9	3	1	2	4	6	5				1	7	5	8	9	4	2	6	3
4	5	1	6	9	7	2	8	3				4	9	6	1	3	2	8	7	5
3	2	6	5	8	4	7	1	9				3	2	8	6	7	5	4	9	1
1	8	4	7	2	5	3	9	6				2	8	1	4	5	6	7	3	9
9	3	5	4	6	8	1	2	7				7	5	4	3	1	9	6	2	8
2	6	7	1	3	9	8	5	4				6	3	9	2	8	7	1	5	4
6	9	3	8	4	1	5	7	2	6	8	1	9	4	3	7	6	1	5	8	2
7	1	2	9	5	3	6	4	8	9	2	3	5	1	7	9	2	8	3	4	6
5	4	8	2	7	6	9	3	1	7	5	4	8	6	2	5	4	3	9	1	7
						3	8	9	1	7	2	4	5	6						
						4	5	7	3	6	8	2	9	1						
						2	1	6	5	4	9	7	3	8						
1	5	4	8	9	2	7	6	3	8	9	5	1	2	4	5	6	7	9	8	3
7	9	8	3	6	4	1	2	5	4	3	7	6	8	9	2	3	1	5	7	4
6	2	3	5	7	1	8	9	4	2	1	6	3	7	5	4	9	8	1	6	2
9	7	1	2	4	8	3	5	6				7	5	1	3	2	4	6	9	8
3	4	6	1	5	9	2	7	8				2	9	6	7	8	5	4	3	1
5	8	2	6	3	7	9	4	1				4	3	8	9	1	6	2	5	7
8	6	9	4	2	3	5	1	7				5	1	2	6	7	3	8	4	9
2	1	5	7	8	6	4	3	9				8	4	3	1	5	9	7	2	6
4	3	7	9	1	5	6	8	2				9	6	7	8	4	2	3	1	5

(a) $\tan t$ (b) $\cot t$

(c) $\tan\frac{t}{2}$ (d) None of these

33. The area bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is

(a) 1 sq. unit (b) $\frac{1}{3}$ sq. unit

(c) $\frac{2}{3}$ sq. unit (d) $\frac{4}{3}$ sq. units

34. A company has factories located at each of the two places P and Q. From these locations, a certain commodity is delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 7, 6 and 4 units of the commodity while the weekly production capacities of the factories at P and Q are respectively 9 and 8 units. The cost of transportation per unit is as given in the table :

To From	Cost (in ₹)		
	A	**B**	**C**
P	16	10	15
Q	10	12	10

How many units should be transported from each factory to depot *A*, *B*, *C* respectively in order that the transportation cost is minimum?

	P	Q		P	Q
(a)	0, 6, 3	7, 0, 1	(b)	1, 5, 3	6, 0, 2
(c)	0, 5, 4	6, 1, 1	(d)	2, 4, 3	5, 2, 1

Assertion & Reasoning Based MCQs

For question numbers 35-38, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

(a) If both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.

(b) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.

(c) If Statement-I is true but Statement-II is false.

(d) If Statement-I is false and Statement-II is true.

35. Statement-I : The value of

$$\sin\left[\tan^{-1}\left(-\sqrt{3}\right)+\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] \text{ is } 1.$$

Statement-II : $\tan^{-1}(-x) = -\tan^{-1}x$ and $\cos^{-1}(-x) = \cos^{-1}x$.

36. Statement-I : If $A=\frac{1}{3}\begin{bmatrix}1 & -2 & 2\\ -2 & 1 & 2\\ -2 & -2 & -1\end{bmatrix}$, then $A\,(A^T) = I$

Statement-II : For any square matrix *A*, $(A^T)^T = A$

37. Statement-I : The area of the region bounded by the triangle whose vertices are (–1, 0), (1, 3) and (3, 2) using integration, is 4 sq. units.

Statement-II : The area of the triangular region whose sides have the equation $y = 2x + 1$, $y = 3x + 1$ and $x = 4$, is 4 sq. units.

38. Statement-I : An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. Then, the probability that the second ball is red is $\frac{1}{2}$.

Statement-II : A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Then, the probability that the ball is drawn form the first bag is $\frac{2}{3}$.

Match the Following

39. Match the column I with column II and select the correct answer using the options given below :

	Column I		Column II
(P)	If [·] denotes the greatest integer function and $f(x)=\begin{cases}3[x]-\frac{5\lvert x\rvert}{x}; & x\neq 0\\ 2 \quad ; & x=0\end{cases}$ then $\int_{-3/2}^{2} f(x)\,dx$ is a/an	(1)	Positive integer
(Q)	The value of definite integral $\int_{-\pi/2}^{\pi/2}\frac{\cos x}{1+e^x}dx$ is a/an	(2)	Real number

(R)	If $I_1 = \int_1^{\sin\theta} \frac{x}{1+x^2}dx$ and $I_2 = \int_1^{\text{cosec}\theta} \frac{1}{x(x^2+1)}dx$, then the value of $\begin{vmatrix} I_1 & I_1^2 & I_2 \\ e^{I_1+I_2} & I_2^2 & -1 \\ 1 & I_1^2+I_2^2 & -1 \end{vmatrix}$ is a/an	(3)	Integer
(S)	Let $f(x)$ be a polynomial of degree 2 satisfying $f(0) = 1$, $f'(0) = -2$ and $f''(0) = 6$, then $\int_{-c}^{-b} f(x)dx$ is a/an	(4)	Rational number
		(5)	Negative integer

	P	Q	R	S
(a)	2, 3	1, 2	3, 4	3, 4
(b)	2	1, 2, 3	2, 3	1, 2, 3
(c)	1, 4	2, 3	2	2, 3, 4
(d)	3, 4	1, 2	1	1, 2, 3

40. Match Column I with Column II and select the correct answer using the given options.

	Column I		Column II
(P)	Lines $\frac{x-1}{-2} = \frac{y+2}{3} = \frac{z}{-1}$ and $\vec{r} = (3\hat{i} - \hat{j} + \hat{k}) + t(\hat{i} + \hat{j} + \hat{k})$ are	(1)	intersecting
(Q)	Lines $\frac{x+5}{1} = \frac{y-3}{7} = \frac{z+3}{3}$ and $x - y + 2z - 4 = 0 = 2x + y - 3z + 5$ are	(2)	perpendicular
(R)	Lines $(x = t - 3,\ y = -2t + 1,\ z = -3t - 2)$ and $\vec{r} = (t+1)\hat{i} + (2t+3)\hat{j} + (-t-9)\hat{k}$ are	(3)	parallel
(S)	Lines $\vec{r} = (\hat{i} + 3\hat{j} - \hat{k}) + t(2\hat{i} - \hat{j} - \hat{k})$ and $\vec{r} = (-\hat{i} - 2\hat{j} + 5\hat{k}) + s(\hat{i} - 2\hat{j} + \frac{3}{4}\hat{k})$ are	(4)	coplanar

	P	Q	R	S
(a)	2	3	2, 1, 4	1, 4
(b)	3, 4	1	1,2	4
(c)	4, 3, 1	2	4,2	3

(d) None of these

Case Based MCQs

Case I : Read the following and answer any four questions from 41 to 45 given below.

Two schools *A* and *B* want to award their selected students on the values of Honesty, Hard work and Punctuality. The school *A* wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of ₹ 2200. School *B* wants to spend ₹ 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school *A*). The total amount of award for one prize on each value is ₹ 1200.

Using the concept of matrices and determinants, answer the following questions.

41. What is the award money for Honesty?
(a) ₹ 350 (b) ₹ 300 (c) ₹ 500 (d) ₹ 400

42. What is the award money for Punctuality?
(a) ₹ 300 (b) ₹ 280 (c) ₹ 450 (d) ₹ 500

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1. (d) **2.** (a) **3.** (b) **4.** (c) **5.** (b)
6. (b) **7.** (d) **8.** (a,b) **9.** (c,d) **10.** (c,d)
11. (d) **12.** (c) **13.** (c, d) **14.** (a) **15.** (a)
16. (c) **17.** (8001) **18.** (10) **19.** (219) **20.** (168)

43. What is the award money for Hard work?

(a) ₹ 500 (b) ₹ 400 (c) ₹ 300 (d) ₹ 550

44. If a matrix P is both symmetric and skew-symmetric, then $|P|$ is equal to

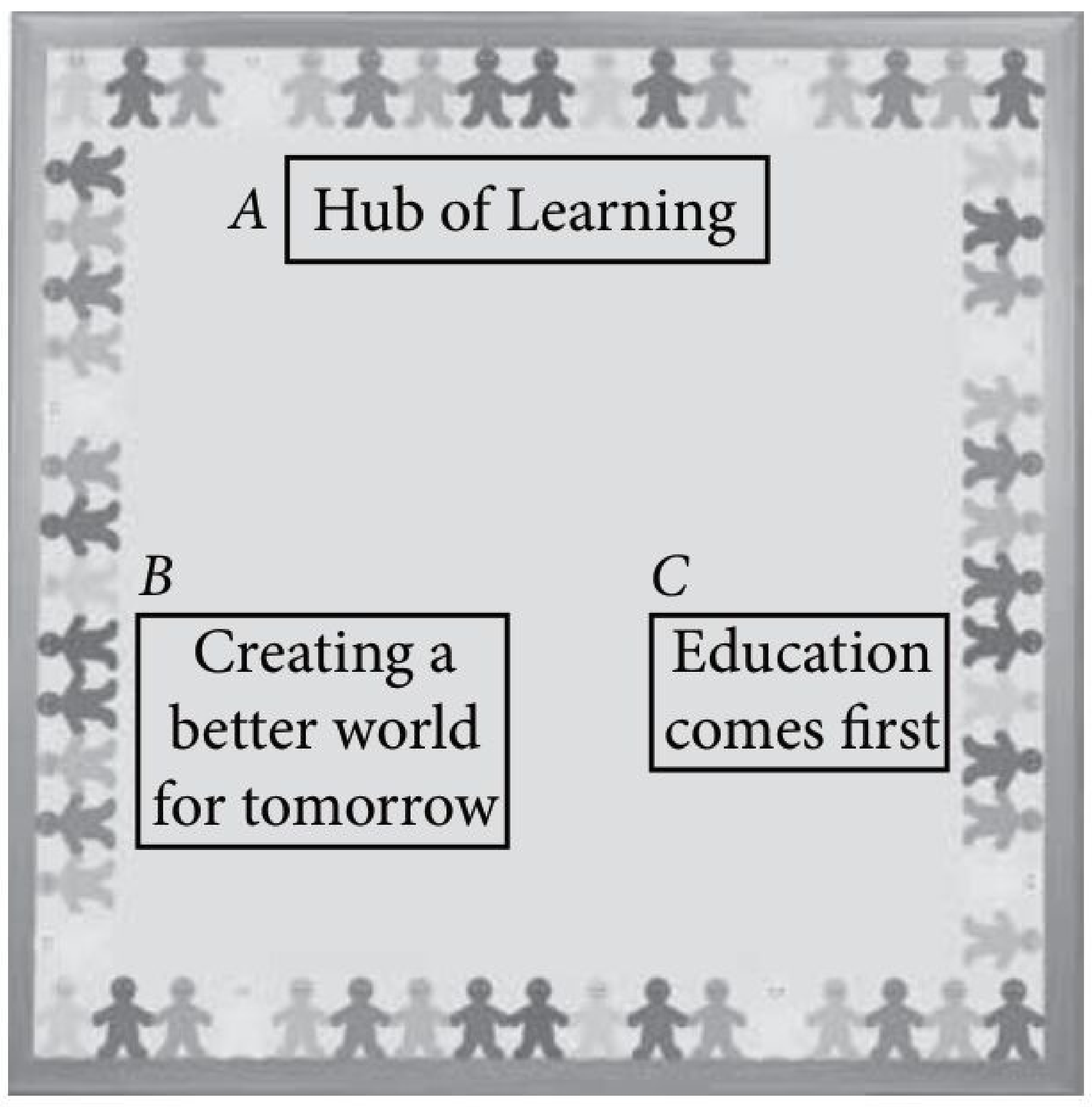
(a) 1 (b) -1
(c) 0 (d) none of these

45. If P and Q are two matrices such that $PQ = Q$ and $QP = P$, then $|Q^2|$ is equal to

(a) $|Q|$ (b) $|P|$ (c) 1 (d) 0

Case II : Read the following passage and answer the questions from 46 to 50.

Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, −3, −2) and (−2, 2, 6) respectively.



Based on the above information, answer the following questions.

46. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points A, B and C respectively, then $\vec{a}+\vec{b}+\vec{c}$ is equal to

(a) $2\hat{i}+3\hat{j}+6\hat{k}$ (b) $2\hat{i}-3\hat{j}-6\hat{k}$

(c) $2\hat{i}+8\hat{j}+3\hat{k}$ (d) $2(7\hat{i}+8\hat{j}+3\hat{k})$

47. Which of the following is not true?

(a) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$ (b) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\vec{0}$

(c) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA}=\vec{0}$ (d) $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\vec{0}$

48. Area of ΔABC is

(a) 19 sq. units (b) $\sqrt{1937}$ sq. units

(c) $\frac{1}{2}\sqrt{1937}$ sq. units (d) $\sqrt{1837}$ sq. units

49. Suppose, if the given slogans are to be placed on a straight line, then the value of $|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$ will be equal to

(a) -1 (b) -2 (c) 2 (d) 0

50. If $\vec{a}=2\hat{i}+3\hat{j}+6\hat{k}$, then unit vector in the direction of vector $\vec{a}$ is

(a) $\frac{2}{7}\hat{i}-\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}$ (b) $\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$

(c) $\frac{3}{7}\hat{i}+\frac{2}{7}\hat{j}+\frac{6}{7}\hat{k}$ (d) None of these

ANSWER KEYS

1.	(c)	**2.**	(c)	**3.**	(d)	**4.**	(b)	**5.**	(c)
6.	(c)	**7.**	(c)	**8.**	(d)	**9.**	(b)	**10.**	(a)
11.	(b)	**12.**	(c)	**13.**	(b)	**14.**	(a)	**15.**	(b)
16.	(d)	**17.**	(a)	**18.**	(c)	**19.**	(c)	**20.**	(b)
21.	(b)	**22.**	(c)	**23.**	(d)	**24.**	(c)	**25.**	(b)
26.	(c)	**27.**	(c)	**28.**	(a)	**29.**	(c)	**30.**	(a)
31.	(d)	**32.**	(b)	**33.**	(d)	**34.**	(a)	**35.**	(c)
36.	(b)	**37.**	(c)	**38.**	(b)	**39.**	(b)	**40.**	(a)
41.	(b)	**42.**	(d)	**43.**	(b)	**44.**	(c)	**45.**	(a)
46.	(a)	**47.**	(c)	**48.**	(c)	**49.**	(d)	**50.**	(b)

* * *

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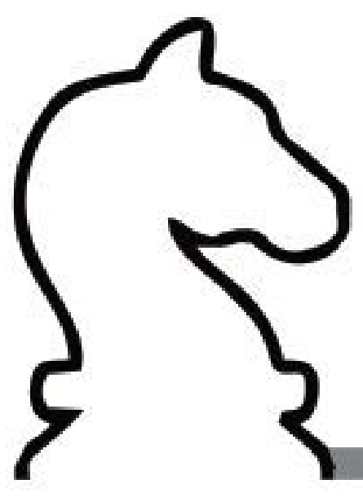
Challenging PROBLEMS

ON

Sequences and Series



1. The first term of a finite G.P. of real numbers is positive, and the sum of the series is negative, then a possible number of terms in the series is

(a) 20 (b) 21 (c) 23 (d) 25

2. $\sum_{k=1}^{100} (-1)^k k^2 =$

(a) 5000 (b) 5050 (c) 6000 (d) 6050

3. Simplify : $(0^3 - 350)(1^3 - 349)(2^3 - 348) \ldots\ldots (349^3 - 1)(350^3 - 0)$

(a) 2 (b) 1 (c) 0 (d) 5

4. The sum of the infinite series

$1+\frac{1}{2}+\frac{1}{10}+\frac{1}{20}+\frac{1}{100}+\ldots\ldots$ is

(a) $\frac{5}{2}$ (b) $\frac{5}{3}$ (c) $\frac{5}{4}$ (d) 1

5. Let $p=\sum_{k=1}^{\infty}\frac{1}{k^2}, q=\sum_{k=1}^{\infty}\frac{1}{k^3}$, then $\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\frac{1}{(j+k)^3}=$

(a) $p+q$ (b) $p-q$ (c) pq (d) p/q

6. Set A consists of m consecutive integers whose sum is $2m$ and set B consists of $2m$ consecutive integers whose sum is m. The difference between the greatest element of A and the greatest element of B is 1051, then $m =$

(a) 2105 (b) 1025 (c) 5012 (d) 2501

7. Given that $x_1 = 211, x_2 = 375, x_3 = 420, x_4 = 523$ and $x_n = x_{n-1} - x_{n-2} + x_{n-3} - x_{n-4}, n \geq 5$, then $x_{531} + x_{753} + x_{975} =$

(a) 898 (b) 1010 (c) 1321 (d) 1430

8. Let the sequence of integers $a_1, a_2, \ldots\ldots$ be defined by $a_1 = 2, a_2 = 3, a_3 = 7, a_4 = 43$ etc. In general $a_{n+1} = 1 + a_1a_2 \ldots\ldots a_n, n \geq 1$. If $S_n = \frac{1}{a_1}+\frac{1}{a_2}+\ldots.+\frac{1}{a_n}$ and $P_n = \frac{1}{a_1a_2a_3\ldots.a_n}$, then $S_{21} + P_{21} =$

(a) 1 (b) 21 (c) 20 (d) $\frac{1}{20}$

9. Define two sequences of rational numbers as follows. Let $a_0 = 2, b_0 = 3$ and $a_{n+1}=\frac{a_n^2}{b_n}$ and $b_{n+1}=\frac{b_n^2}{a_n}, n \geq 0$.

If $b_8 = \frac{3^p}{2^q}$, then $p - q =$

(a) 0 (b) 1 (c) –1 (d) 2

10. The sequence $a_0, a_1, \ldots$ satisfies $a_n = ka_{n-1} - a_{n-2}$, $n \geq 2$. Then sequence $\langle a_n \rangle$ is periodic for $k=\sqrt{2}$ with period

(a) 7 (b) 8 (c) 14 (d) 16

11. Sum $S=\sum_{n=1}^{\infty}\frac{(7n+32)\cdot 3^n}{n(n+2)\cdot 4^n}$ is

(a) $\frac{31}{2}$ (b) $\frac{32}{2}$ (c) $\frac{33}{2}$ (d) $\frac{34}{2}$

12. Let $H_n = 1+\frac{1}{2}+\frac{1}{3}+\ldots..+\frac{1}{n}$ and $T_n = \frac{1}{(n+1)H_nH_{n+1}}$, then $T_1 + T_2 + T_3 + \ldots. \infty =$

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

13. Let a, b be fixed +ve integers. The sequence $\langle x_n \rangle$ is given by the recursive relation

$x_{n+1} = x_n + a + \sqrt{b^2 + 4ax_n}, n \geq 0$ and $x_0 = 0$.

Then $\lim_{n\to\infty}\frac{x_n}{n^2} =$

(a) a (b) b (c) $a + b$ (d) ab

14. Sum $S=\sum_{k=1}^{\infty}\frac{k^3+k}{2^k}$ is

(a) 21 (b) 24 (c) 26 (d) 28

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15. The sequence $<a_n>$ satisfies $a_1 = 1$ and $5^{a_{n+1}-a_n} = \frac{3}{3n+2} + 1$ for $n \ge 1$. The least integer m (> 1) so that a_m is an integer is

(a) 25 (b) 31 (c) 41 (d) 52

16. For any positive integer n, let $\overline{n}$ denote the closest integer to $\sqrt{n}$, then $\sum_{n=1}^{\infty} \frac{2^{\overline{n}} + \left(\frac{1}{2}\right)^{\overline{n}}}{2^n} =$

(a) 2 (b) 3 (c) 1/2 (d) 1/3

17. The value of $\sum_{k=1}^{\infty} \frac{6^k}{(3^k - 2^k)(3^{k+1} - 2^{k+1})}$ is

(a) 2 (b) 1/2 (c) 3 (d) 1/3

18. Let a_0, a_1, a_2,.....,a_n be a sequence of numbers satisfying $(3 - a_{n+1})(6 + a_n) = 18$ and $a_0 = 3$.

If $\frac{1}{a_0} + \frac{1}{a_1} + ... + \frac{1}{a_n} = \frac{2^{n+2} - n - 3}{\lambda}$, then $\lambda =$

(a) 1 (b) 2 (c) 3 (d) 4

19. If $\sum_{r=1}^{20} \left(\frac{r^2 - \frac{1}{2}}{r^4 + \frac{1}{4}} \right) = \frac{\lambda}{\mu}$, then $\mu - \lambda - 20 =$ ______

(λ, μ being in lowest terms)

(a) 20 (b) 21 (c) 22 (d) 23

20. $a, b, c \in R^+$, the least value of $\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1+c}{2a}$ is

(a) 11/4 (b) 21/4 (c) 30/4 (d) 5/4

SOLUTIONS

1. (a) : $S = \frac{a(r^n - 1)}{r - 1}$. If $r \ge 0$, then every term is non-negative and hence S is non-negative *i.e.*, a contradiction to given data. Hence, r must be negative ⇒ $r^n - 1$ must be +ve *i.e.*, n should be even.

2. (b) :

$$\sum_{k=1}^{100} (-1)^k \cdot k^2 = (2^2 - 1^2) + (4^2 - 3^2) + ... + (100^2 - 99^2)$$

$= (2 + 1) + (4 + 3) + ... + (100 + 99)$

$= 1 + 2 + 3 + 4 + ... + 100 = 5050.$

3. (c) : Notice that 8th term is 0.

4. (b) : Given, $\text{sum} = \left(1 + \frac{1}{10} + \frac{1}{100} + ...\right) + \left(\frac{1}{2} + \frac{1}{20} + \frac{1}{200} + ...\right) = \frac{5}{3}.$

5. (b) : Required sum $= \sum_{j=1}^{\infty} \left(\frac{1}{(j+1)^3} + \frac{1}{(j+2)^3} + ... \right)$

$$= \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + ...$$
$$+ \frac{1}{3^3} + \frac{1}{4^3} + ...$$
$$+ \frac{1}{4^3} + \frac{1}{5^3} + ...$$

$$= \sum_{n=2}^{\infty} \frac{n-1}{n^3} = \sum_{n=2}^{\infty} \frac{1}{n^2} + 1 - \sum_{n=2}^{\infty} \frac{1}{n^3} - 1 = p - q$$

6. (a) : Set $A = \{a, a + 1, ..., a + m - 1\}$

$\therefore$ Sum, $2m = \frac{m}{2}[2a + (m - 1)]$ *i.e.*, $a = \frac{5 - m}{2}$

Also, set $B = \{b, b + 1, ... b + 2m - 1\}$

Now sum, $m = \frac{2m}{2}[2b + (2m - 1)]$ *i.e.*, $b = 1 - m$

$A_{\max} = a + m - 1 = \frac{m+3}{2}$, $B_{\max} = b + 2m - 1 = m$

According to the question, $\left| \frac{m+3}{2} - m \right| = 1051$

⇒ $m = 2105.$

7. (a) : $x_5 = 267$. Iterating n to $(n - 1)$ and simplifying, we have $x_n = -x_{n-5}$ *i.e*, $x_n = (-1)^k x_{n-5k}$

Hence, $x_{531} = (-1)^{106} x_1$, $x_{753} = (-1)^{150} x_3$, $x_{975} = (-1)^{194} x_5$

8. (a) : Let $t_n = S_n + P_n$, then $t_1 = 1$, $t_2 = 1$, $t_3 = 1$, ... So, a general guess is $t_n = 1$.

Indeed $t_n = S_n + P_n = S_{n+1} + P_{n+1} = t_{n+1}$

Hence, $t_{n+1} = t_n = t_{n-1} = ... = t_1 = 1$

i.e., $S_{21} + P_{21} = t_{21} = 1$

9. (b) : $a_{n+1} b_{n+1} = a_n b_n = a_{n-1} b_{n-1} = ... = a_0 b_0 = 6$

Hence, $b_{n+1} = \frac{b_n^3}{6}$

So, $b_8 = \frac{b_0^{3^8}}{6^{3^7} \cdot 6^{3^6} .... 6^{3^2} \cdot 6^{3^1} \cdot 6} = \frac{3^{6561}}{6^{3280}} = \frac{3^{3281}}{2^{3280}}$

$\therefore$ $p - q = 3281 - 3280 = 1$

10. (b) : With $k = \sqrt{2}$, $a_2 = \sqrt{2}a_1 - a_0$, $a_3 = a_1 - \sqrt{2}a_0$

Similarly, $a_8 = a_0$, $a_9 = a_1$

$\therefore$ Period = 8

11. (c) : Using partial fractions, given sum S is

$$S = \sum_{n=1}^{\infty} \left(\frac{4^2}{n} - \frac{3^2}{n+2} \right) \cdot \frac{3^n}{4^n}$$

Now, notice that $\frac{-3^2}{k+2} \cdot \frac{3^k}{4^k} = \frac{-4^2}{k+2} \cdot \frac{3^{k+2}}{4^{k+2}}$

Hence, terms in S, cancels after two-two terms each.

$\therefore\ S=\frac{1}{1}\cdot 3\cdot 4+\frac{1}{2}\cdot 3\cdot 3=\frac{33}{2}$

12. (a) : Notice that $H_{n+1}=H_n+\frac{1}{n+1}$

Hence, $T_n=\frac{1}{H_n}-\frac{1}{H_{n+1}}$

So, $T_1+T_2+T_3+\dots\dots\dots\infty=\frac{1}{H_1}=1$

13. (a) : The given relation can be re-written as

$$\sqrt{b^2+4ax_{n+1}}=2a+\sqrt{b^2+4ax_n}$$

i.e., $y_{n+1}=2a+y_n$, where $y_n=\sqrt{b^2+4ax_n}$

i.e., y_n is an A.P. series with common difference = $2a$

Hence, $x_n=an^2+bn$.

14. (d) : Given $S=\sum_{k=1}^{\infty}\frac{k^3+k}{2^k}=2$

$$+\sum_{k=1}^{\infty}\frac{(k+1)^3+(k+1)-k^3-k}{2^k}$$

$$S=2+\sum_{k=1}^{\infty}\frac{3k^2+3k+2}{2^k}=2+3\sum\frac{k^2}{2^k}+3\sum\frac{k}{2^k}+2\sum\frac{1}{2^k}$$

Now, $\sum_{k=1}^{\infty}kx^k=\frac{x}{(1-x)^2}$ and $\sum_{k=1}^{\infty}k^2x^k=\frac{x(x+1)}{(1-x)^3}$

Putting $x=1/2$ in these results

$$\sum\frac{k}{2^k}=2,\ \sum\frac{k^2}{2^k}=6 \text{ and } \sum\frac{1}{2^k}=1$$

Hence, $S=2+3(6)+3(2)+2(1)=28$.

15. (c) : $5^{a_{n+1}-a_n}=\frac{3n+5}{3n+2},\ 5^{a_n-a_{n-1}}=\frac{3n+2}{3n-1},\dots.$

Multiplying all, $5^{a_{n+1}-a_1}=\frac{3n+5}{5}\Rightarrow a_n=\log_5(3n+2)$

So, $a_m=\log_5(3m+2)$ = integer for least $m=41$.

16. (b) : Notice that $\left(r-\frac{1}{2}\right)^2=r^2-r+\frac{1}{4}$

and $\left(r+\frac{1}{2}\right)^2=r^2+r+\frac{1}{4}$

So $\overline{n}=r$ iff $r^2-r+1\le n\le r^2+r+1$

Hence, required sum is

$$\sum_{r=1}^{\infty}\sum_{\overline{n}=r}\frac{2^{\overline{n}}+\left(\frac{1}{2}\right)^{\overline{n}}}{2^n}=\sum_{r=1}^{\infty}\sum_{n=r^2-r+1}^{r^2+r+1}\frac{2^r+2^{-r}}{2^n}=3$$

17. (a) : Given expression can be expressed as

$$\sum_{k=1}^{\infty}\left(\frac{3^k}{3^k-2^k}-\frac{3^{k+1}}{3^{k+1}-2^{k+1}}\right)$$

18. (c) : As required sum is of the form $\frac{1}{a_n}$, lets replace $\frac{1}{a_n}$ with b_n to get the new recurrence relation,

$3b_{n+1}-6b_n=1\ \Rightarrow\ b_{n+1}+\frac{1}{3}=2\left(b_n+\frac{1}{3}\right)$

Let $b_n+\frac{1}{3}=c_n$, then $c_{n+1}=2c_n$

(G.P. with common ratio 2)

Hence, $b_n=\frac{1}{3}(2^{n+1}-1)$

and so, $\sum_{r=0}^{n}\frac{1}{a_r}=\frac{1}{3}(2^{n+2}-n-3)$ *i.e.*, $\lambda=3$

19. (b) : $T_r=\frac{r^2-\frac{1}{2}}{r^4+\frac{1}{4}}=\frac{r-\frac{1}{2}}{r^2-r+\frac{1}{2}}-\frac{(r+1)-\frac{1}{2}}{(r+1)^2-(r+1)+\frac{1}{2}}$

Hence, $\sum_{r=1}^{20}T_r=\sum_{r=1}^{20}(S_r-S_{r+1})=S_1-S_{21}=1-\frac{41}{841}=\frac{800}{841}$

$\therefore\ \mu-\lambda-20=21$ $\left(\text{where } S_r=\frac{r-\frac{1}{2}}{r^2-r+\frac{1}{2}}\right)$

20. (d) : Apply A.M. ≥ G.M. on the quantities

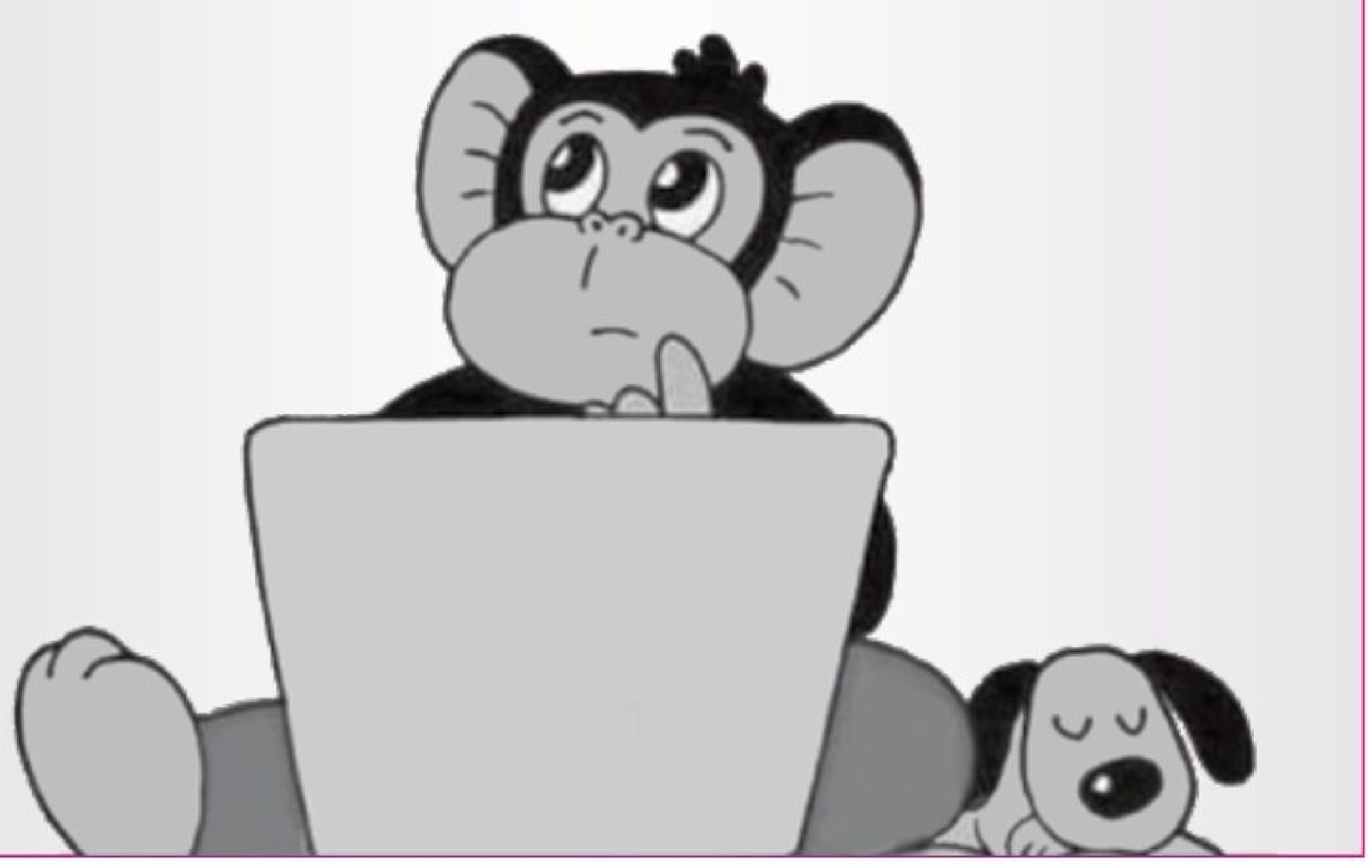
$\frac{a^3}{4b},\frac{b}{8c^2},\frac{1}{2a},\frac{c}{4a},\frac{c}{4a}$.

◆◆

COMIC CAPSULE

Hindi : Rishtedar ke yaha shaadi hai.

Maths : Relation me function hai.



CONCEPT MAP

TRIGONOMETRIC FUNCTIONS

Class XI

Relationship between Degree and Radian

- 1 radian = $\dfrac{180^\circ}{\pi}$ = 57°16′ (approximately)
- $1^\circ = \dfrac{\pi}{180}$ radian = 0.01746 radian (approximately)

Sum and Difference of Two Angles

- $\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$
- $\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$
- $\tan(\theta \pm \phi) = \dfrac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\cdot\tan\phi}$
- $\cot(\theta \pm \phi) = \dfrac{\cot\theta\cdot\cot\phi \mp 1}{\cot\phi \pm \cot\theta}$
- $\sin\theta + \sin\phi = 2\sin\left(\dfrac{\theta+\phi}{2}\right)\cos\left(\dfrac{\theta-\phi}{2}\right)$
- $\sin\theta - \sin\phi = 2\cos\left(\dfrac{\theta+\phi}{2}\right)\sin\left(\dfrac{\theta-\phi}{2}\right)$
- $\cos\theta + \cos\phi = 2\cos\left(\dfrac{\theta+\phi}{2}\right)\cos\left(\dfrac{\theta-\phi}{2}\right)$
- $\cos\theta - \cos\phi = -2\sin\left(\dfrac{\theta+\phi}{2}\right)\sin\left(\dfrac{\theta-\phi}{2}\right)$
- $2\sin\theta\cos\phi = \sin(\theta+\phi) + \sin(\theta-\phi)$
- $2\cos\theta\sin\phi = \sin(\theta+\phi) - \sin(\theta-\phi)$
- $2\cos\theta\cos\phi = \cos(\theta+\phi) + \cos(\theta-\phi)$
- $2\sin\theta\sin\phi = \cos(\theta-\phi) - \cos(\theta+\phi)$

Trigonometric Equations

- $\sin\theta = 0 \Rightarrow \theta = n\pi, n \in Z$
- $\cos\theta = 0 \Rightarrow \theta = (2n+1)\pi/2, n \in Z$
- $\tan\theta = 0 \Rightarrow \theta = n\pi, n \in Z$
- $\cot\theta = 0 \Rightarrow \theta = (2n+1)\pi/2, n \in Z$
- $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha, n \in Z$
- $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in Z$
- $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha, n \in Z$

Functions	Domain	Range
$\sin\theta$	R	$[-1, 1]$
$\cos\theta$	R	$[-1, 1]$
$\tan\theta$	$R - \left\{(2n+1)\dfrac{\pi}{2}, n \in Z\right\}$	R
$\sec\theta$	$R - \left\{(2n+1)\dfrac{\pi}{2}, n \in Z\right\}$	$R - (-1, 1)$
$\operatorname{cosec}\theta$	$R - \{n\pi, n \in Z\}$	$R - (-1, 1)$
$\cot\theta$	$R - \{n\pi, n \in Z\}$	R

Multiple Angles

- $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$
  $= 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$
- $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
- $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
- $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
- $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

Properties of Triangles

- **Sine rule:** $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$
- **Cosine rule:** $a^2 = b^2 + c^2 - 2bc\cos A$, $b^2 = c^2 + a^2 - 2ca\cos B$, $c^2 = a^2 + b^2 - 2ab\cos C$
- **Napier's rule:**

$$\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\frac{A}{2},$$

$$\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right)\cot\frac{B}{2},$$

$$\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$$

DETERMINANTS

Class XII

CONCEPT MAP

Determinant

Corresponding to every square matrix A, there exists a number called the determinant of A and denoted by |A|.

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Then,

$$|A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Properties

- The value of a determinant remains unaltered if its rows and columns are interchanged.
- If two rows (or columns) of a determinant are interchanged, the value of the determinant is multiplied by –1.
- If any two rows (or columns) of a determinant are identical, then the value of determinant is zero.
- If the elements of a row (or column) of a determinant are multiplied by any scalar, then the value of the new determinant is equal to same scalar times the value of the original determinant.
- If each element of any row (or column) of a determinant is the sum of two numbers, then the determinant is expressible as the sum of two determinants of the same order.

Minors and Cofactors

- For any matrix $A = [a_{ij}]_{n \times n}$, if we leave the row and the column of the element a_{ij}, then the value of determinant thus obtained is called the minor of a_{ij} and it is denoted by M_{ij}.
- The minor M_{ij} multiplied by $(-1)^{i+j}$ is called the cofactor of the element a_{ij} and denoted by A_{ij}.

$\therefore\ A_{ij} = (-1)^{i+j} M_{ij}$

Singular and Non-Singular Matrices

A square matrix A of order n is said to be

- Singular if $|A| = 0$
- Non-singular if $|A| \neq 0$

Note : If A and B are non-singular matrices of the same order, then AB and BA are also non-singular matrices of the same order.

Inverse of a Matrix

For any square matrix A, inverse of A is defined as

$$A^{-1} = \frac{1}{|A|}(\text{adj } A), |A| \neq 0$$

Properties

- $(A^{-1})^{-1} = A$
- $(A')^{-1} = (A^{-1})'$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

Adjoint of a Matrix

Let $B = [A_{ij}]$ be the matrix of cofactors of matrix $A = [a_{ij}]$. Then the transpose of B is called the adjoint of matrix A.

Properties of adj (A)

If A is non-singular matrix of order n, then

- $A(\text{adj } A) = (\text{adj } A)A = |A|\,I_n$
- $|A \text{ adj } A| = |A|^n$
- $\text{adj }(AB) = (\text{adj } B).(\text{adj } A)$
- $|\text{adj } A| = |A|^{n-1}$
- $\text{adj }(\text{adj } A) = |A|^{n-2}A$
- $|\text{adj }(\text{adj } A)| = |A|^{(n-1)^2}$

Solution of System of Linear Equations

Let $AX = B$ be the given system of equations :

- If $|A| \neq 0$, the system is consistent and has a unique solution.
- If $|A| = 0$ and $(\text{adj } A)B \neq O$, then the system is inconsistent and hence it has no solution.
- If $|A| = 0$ and $(\text{adj } A)B = O$, then the system may be either consistent or inconsistent according as the system has either infinitely many solutions or no solution.

Rank of a Matrix

- The rank of a matrix is the maximum number of independent rows (or columns). A square matrix of order n is non-singular if and only if its rank is equal to n.

Properties :

- Rank of the sum of two matrices is the sum of their ranks.
- Rank of the product of two matrices is the rank of either matrix.
- Rank of a matrix whose each and every element is unit, is 1.

Area of a Triangle

Let ABC be a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then area of ΔABC is

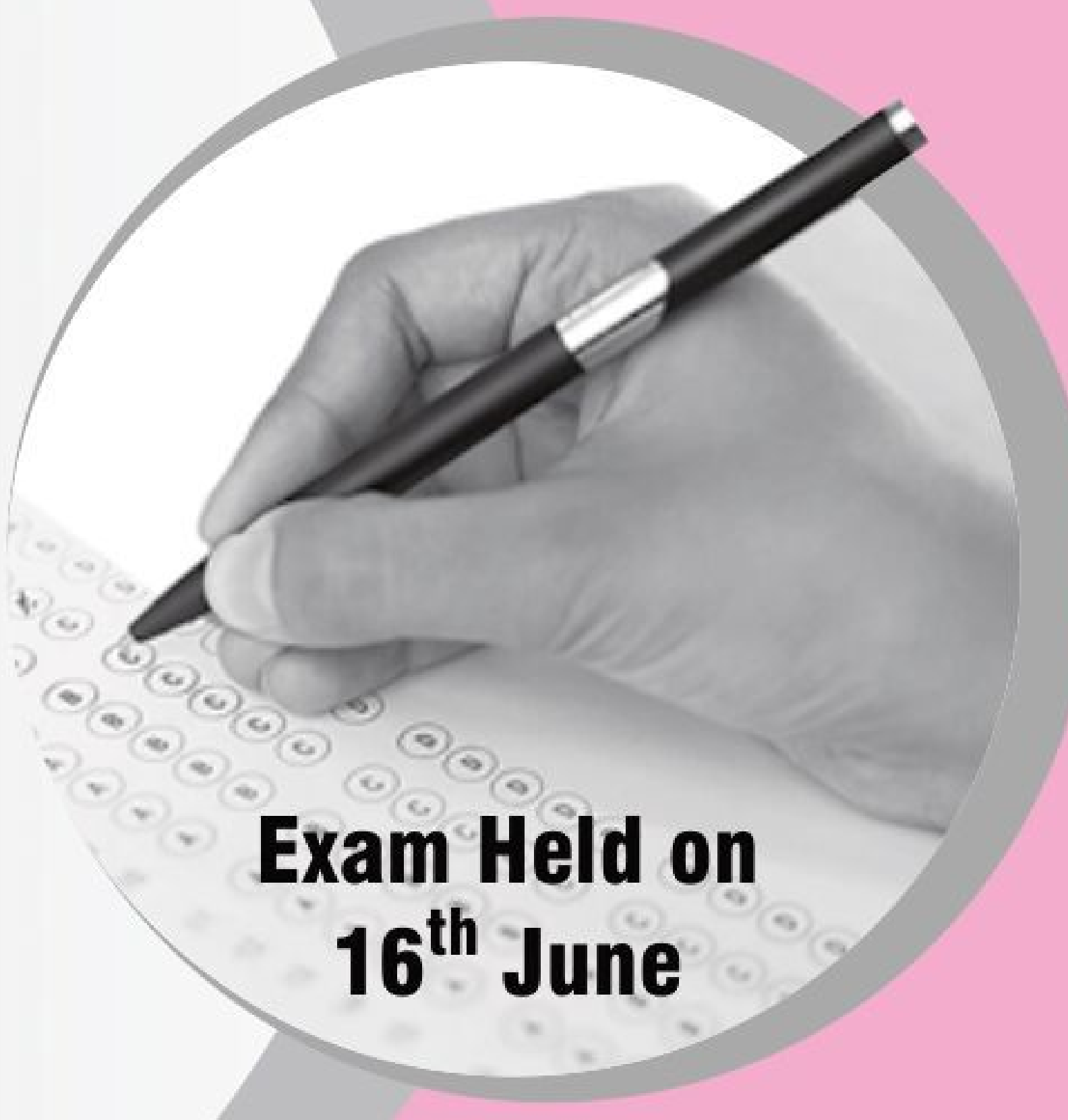
$$\Delta = \frac{1}{2}\left\|\begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix}\right\|$$

Points to be Remember on Area of Triangle

- Area is a positive quantity, we always take the absolute value of the determinant.
- If area is given, use both positive and negative values of the determinant for calculation.
- The area of the triangle formed by three collinear points is zero.

SOLVED PAPER **2022**

Karnataka CET



Exam Held on 16th June

1. If all permutations of the letters of the word MASK are arranged in the order as in dictionary with or without meaning which one of the following is 19th word?
(a) KAMS (b) AKMS (c) SAMK (d) AMSK

2. If $a_1, a_2, a_3, ..., a_{10}$ is a geometric progression and $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals
(a) $3(5^2)$ (b) 5^3 (c) 5^4 (d) $2(5^2)$

3. If the straight line $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals
(a) -5 (b) 29 (c) 5 (d) -29

4. The octant in which the point $(2, -4, -7)$ lies is
(a) Eighth (b) Fourth (c) Third (d) Fifth

5. If $f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \le x < 3 \end{cases}$,

the quadratic equation whose roots are $\lim\limits_{x \to 2^-} f(x)$ and $\lim\limits_{x \to 2^+} f(x)$ is
(a) $x^2 - 14x + 49 = 0$ (b) $x^2 - 6x + 9 = 0$
(c) $x^2 - 10x + 21 = 0$ (d) $x^2 - 7x + 8 = 0$

6. If $3x + i(4x - y) = 6 - i$ where x and y are real numbers, then the values of x and y are respectively,
(a) 3, 9 (b) 2, 9 (c) 2, 4 (d) 3, 4

7. If the standard deviation of the numbers $-1, 0, 1, k$ is $\sqrt{5}$ where $k > 0$, then k is equal to
(a) $4\sqrt{\frac{5}{3}}$ (b) $2\sqrt{\frac{10}{3}}$ (c) $\sqrt{6}$ (d) $2\sqrt{6}$

8. If the set x contains 7 elements and set y contains 8 elements, then the number of bijections from x to y is
(a) 0 (b) 7! (c) ${}^8P_7$ (d) 8!

9. If $f: R \to R$ be defined by $f(x) = \begin{cases} 2x & : & x > 3 \\ x^2 & : & 1 < x \le 3 \\ 3x & : & x \le 1 \end{cases}$

then $f(-1) + f(2) + f(4)$ is
(a) 5 (b) 9 (c) 10 (d) 14

10. Let the relation R is defined in N by $a\ R\ b$, if $3a + 2b = 27$ then R is
(a) $\{(1, 12)(3, 9)(5, 6)(7, 3)\}$
(b) $\{(1, 12)(3, 9)(5, 6)(7, 3)(9, 0)\}$
(c) $\{\left(0, \frac{27}{2}\right)(1, 12)(3, 9)(5, 6)(7, 3)\}$
(d) $\{(2, 1)(9, 3)(6, 5)(3, 7)\}$

11. $\lim\limits_{y \to 0} \frac{\sqrt{3 + y^3} - \sqrt{3}}{y^3} =$
(a) $\frac{1}{2\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $3\sqrt{2}$

12. If A is a matrix of order 3×3, then $(A^2)^{-1}$ is equal to
(a) $(-A^2)^2$ (b) A^2 (c) $(A^{-1})^2$ (d) $(-A)^{-2}$

13. If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, then the inverse of the matrix A^3 is
(a) A (b) 1 (c) -1 (d) $-A$

14. If A is a skew symmetric matrix, then A^{2021} is
(a) Row matrix
(b) Symmetric matrix
(c) Column matrix
(d) Skew symmetric matrix

15. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then $(aI + bA)^n$ is (where I is the identity matrix of order 2)
(a) $a^2 I + a^{n-1} b \cdot A$ (b) $a^n I + n a^n b A$
(c) $a^n I + n a^{n-1} b A$ (d) $a^n I + b^n A$

16. If A is a 3×3 matrix such that $|5 \cdot \text{adj } A| = 5$ then $|A|$ is equal to

(a) ± 1 (b) $\pm 1/5$ (c) $\pm 1/25$ (d) ± 5

17. If there are two values of 'a' which makes determinant $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

Then the sum of these numbers is

(a) -4 (b) 4 (c) 9 (d) 5

18. If the vertices of a triangle are $(-2, 6)$, $(3, -6)$ and $(1, 5)$, then the area of the triangle is

(a) 40 sq. units (b) 30 sq. units
(c) 15.5 sq. units (d) 35 sq. units

19. Domain of $\cos^{-1} [x]$ is, where [ ] denotes a greatest integer function

(a) $(-1, 2]$ (b) $[-1, 2]$ (c) $(-1, 2)$ (d) $[-1, 2)$

20. If $y = (1 + x^2) \tan^{-1} x - x$ then $\frac{dy}{dx}$ is

(a) $2x \tan^{-1}x$ (b) $x^2 \tan^{-1}x$
(c) $\frac{\tan^{-1} x}{x}$ (d) $x \tan^{-1}x$

21. If $x = e^{\theta} \sin\theta$, $y = e^{\theta} \cos\theta$ where θ is a parameter, then $\frac{dy}{dx}$ at $(1, 1)$ is equal to

(a) 0 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{4}$

22. If $y = e^{\sqrt{x\sqrt{x\sqrt{x}}}\cdots}\ x > 1$ then $\frac{d^2y}{dx^2}$ at $x = \log_e 3$ is

(a) 3 (b) 0 (c) 5 (d) 1

23. If $f(1) = 1$, $f'(1) = 3$ then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

(a) 10 (b) 35 (c) 33 (d) 12

24. If $y = x^{\sin x} + (\sin x)^x$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is

(a) $\frac{4}{\pi}$ (b) 1 (c) $\pi \log\frac{\pi}{2}$ (d) $\frac{\pi^2}{2}$

25. If $A_n = \begin{bmatrix} 1-n & n \\ n & 1-n \end{bmatrix}$ then

$|A_1| + |A_2| + \ldots + |A_{2021}| =$

(a) -2021 (b) $(2021)^2$ (c) $-(2021)^2$ (d) 4042

26. The function $f(x) = \log(1 + x) - \frac{2x}{2+x}$ is increasing on

(a) $(-\infty, \infty)$ (b) $(-1, \infty)$ (c) $(\infty, -1)$ (d) $(-\infty, 0)$

27. The co-ordinates of the point on the $\sqrt{x} + \sqrt{y} = 6$ at which the tangent is equally inclined to the axes is

(a) $(4, 4)$ (b) $(9, 9)$ (c) $(1, 1)$ (d) $(6, 6)$

28. The function $f(x) = 4 \sin^3 x - 6 \sin^2 x + 12 \sin x + 100$ strictly

(a) decreasing in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(b) increasing in $\left(\pi, \frac{3\pi}{2}\right)$

(c) decreasing in $\left[0, \frac{\pi}{2}\right]$

(d) decreasing in $\left(\frac{\pi}{2}, \pi\right)$

29. If $[x]$ is the greatest integer function not greater than x then $\int_0^8 [x]\, dx$ is equal to

(a) 28 (b) 29 (c) 30 (d) 20

30. $\int_0^{\pi/2} \sqrt{\sin\theta} \cos^3\theta\, d\theta$ is equal to

(a) $\frac{8}{23}$ (b) $\frac{8}{21}$ (c) $\frac{7}{23}$ (d) $\frac{7}{21}$

31. If $e^y + xy = e$ the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $x = 0$ is equal to

(a) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$ (b) $\left(\frac{1}{e}, \frac{-1}{e^2}\right)$
(c) $\left(\frac{-1}{e}, \frac{-1}{e^2}\right)$ (d) $\left(\frac{-1}{e}, \frac{1}{e^2}\right)$

32. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}\, dx$ is equal to

(a) $2(\sin x - x \cos\alpha) + c$
(b) $2(\sin x - 2x \cos\alpha) + c$
(c) $2(\sin x + x \cos\alpha) + c$
(d) $2(\sin x + 2x \cos\alpha) + c$

33. $\int_0^1 \frac{xe^x}{(2+x)^3}\, dx$ is equal to

(a) $\frac{1}{27} \cdot e - \frac{1}{8}$ (b) $\frac{1}{9} \cdot e + \frac{1}{4}$
(c) $\frac{1}{27} \cdot e + \frac{1}{8}$ (d) $\frac{1}{9} \cdot e - \frac{1}{4}$

34. If $\int \frac{dx}{(x+2)(x^2+1)} = a\log|1+x^2| + b\tan^{-1}x$ $+\frac{1}{5}\log|x+2|+c,$ then

(a) $a=\frac{-1}{10}, b=\frac{2}{5}$ (b) $a=\frac{-1}{10}, b=\frac{-2}{5}$

(c) $a=\frac{1}{10}, b=\frac{2}{5}$ (d) $a=\frac{1}{10}, b=\frac{-2}{5}$

35. Area of the region bounded by the curve $y = \tan x$, the x-axis and the line $x=\frac{\pi}{3}$ is

(a) $\log\frac{1}{2}$ (b) 0 (c) log 2 (d) –log 2

36. Evaluate $\int_2^3 x^2\,dx$ as the limit of a sum

(a) $\frac{72}{6}$ (b) $\frac{25}{7}$ (c) $\frac{53}{9}$ (d) $\frac{19}{3}$

37. $\int_0^{\pi/2} \frac{\cos x \sin x}{1+\sin x}\,dx$ is equal to

(a) log 2 – 1 (b) –log 2

(c) log 2 (d) 1 – log 2

38. If $\frac{dy}{dx}+\frac{y}{x}=x^2,$ then $2y(2) - y(1) =$

(a) $\frac{11}{4}$ (b) $\frac{9}{4}$ (c) $\frac{15}{4}$ (d) $\frac{13}{4}$

39. The solution of the differential equation $\frac{dy}{dx} = (x+y)^2$ is

(a) $\tan^{-1}(x+y) = x+c$
(b) $\cot^{-1}(x+y) = c$
(c) $\tan^{-1}(x+y) = 0$
(d) $\cot^{-1}(x+y) = x+c$

40. If $y(x)$ be the solution of differential equation $x\log x\frac{dy}{dx}+y=2x\log x,$ $y(e)$ is equal to

(a) e (b) 2 (c) 0 (d) $2e$

41. If $|\vec a|=2$ and $|\vec b|=3$ and the angle between $\vec a$ and $\vec b$ is 120°, then the length of the vector $\left|\frac{1}{2}\vec a-\frac{1}{3}\vec b\right|^2$ is

(a) 2 (b) $\frac{1}{6}$ (c) 3 (d) 1

42. If $|\vec a\times\vec b|+|\vec a\cdot\vec b|^2=36$ and $|\vec a|=3$ then $|\vec b|$ is equal to

(a) 9 (b) 4 (c) 36 (d) 2

43. If $\vec\alpha=\hat i-3\hat j, \vec\beta=\hat i+2\hat j-\hat k$ then, express $\vec\beta$ in the form $\vec\beta=\vec\beta_1+\vec\beta_2$ where $\vec\beta_1$ is parallel to $\vec\alpha$ and $\vec\beta_2$ is perpendicular to $\vec\alpha$ then $\vec\beta_1$ is given by

(a) $\frac{5}{8}(\hat i-3\hat j)$ (b) $\hat i-3\hat j$

(c) $\frac{5}{8}(\hat i+3\hat j)$ (d) $\hat i+3\hat j$

44. The sum of the degree and order of the differential equation $(1+y_1^2)^{2/3}=y_2$ is

(a) 4 (b) 5 (c) 6 (d) 7

45. The co-ordinates of foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z = 29$ are

(a) (2, 3, 4) (b) (2, –3, 4)

(c) (2, –3, –4) (d) (–2, –3, 4)

46. The angle between the pair of lines $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ and $\frac{x+1}{1}=\frac{y-4}{4}=\frac{z-5}{2}$ is

(a) $\theta=\cos^{-1}\left[\frac{27}{5}\right]$ (b) $\theta=\cos^{-1}\left[\frac{19}{21}\right]$

(c) $\theta=\cos^{-1}\left[\frac{8\sqrt3}{15}\right]$ (d) $\theta=\cos^{-1}\left[\frac{5\sqrt3}{16}\right]$

47. The corner points of the feasible region of an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5), then the minimum value of $z = 4x + 6y$ occurs at

(a) finite number of points
(b) only one point
(c) infinite number of points
(d) only two points

48. A dietician has to develop a special diet using two foods X and Y. Each packet (containing 30 g) of food. X contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Y contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and atmost 300 units of cholesterol. The corner points of the feasible region are

(a) (2, 72), (40, 15), (15, 20)
(b) (0, 23), (40, 15), (2, 72)
(c) (2, 72), (15, 20), (0, 23)
(d) (2, 72), (40, 15), (115, 0)

49. The distance of the point whose position vector is $(2\hat{i}+\hat{j}-\hat{k})$ from the plane $\vec{r}\cdot(\hat{i}-2\hat{j}+4\hat{k})=4$ is

(a) $\frac{8}{\sqrt{21}}$ (b) $\frac{-8}{\sqrt{21}}$ (c) $8\sqrt{21}$ (d) $\frac{-8}{21}$

50. Find the mean number of heads in three tosses of a fair coin.

(a) 1.5 (b) 2.5 (c) 4.5 (d) 3.5

51. If A and B are two events such that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(A\,|\,B)=\frac{1}{4}$, then $P(A'\cap B')$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{12}$ (c) $\frac{3}{16}$ (d) $\frac{3}{4}$

52. A pandemic has been spreading all over the world. The probabilities are 0.7 that there will be a lockdown, 0.8 that the pandemic is controlled in one month if there is a lockdown and 0.3 that it is controlled in one month if there is no lockdown. The probability that the pandemic will be controlled in one month is

(a) 0.65 (b) 1.46 (c) 1.65 (d) 0.46

53. If A and B are two independent events such that $P(\bar{A})=0.75$, $P(A\cup B)=0.65$, and $P(B) = x$, then find the value of x.

(a) $\frac{5}{14}$ (b) $\frac{9}{14}$ (c) $\frac{8}{15}$ (d) $\frac{7}{15}$

54. Suppose that the number of elements in set A is p, the number of elements in set B is q and the number of elements in $A \times B$ is 7 then $p^2 + q^2 =$ ________.

(a) 50 (b) 42 (c) 51 (d) 49

55. The domain of the function

$$f(x)=\frac{1}{\log_{10}(1-x)}+\sqrt{x+2} \text{ is}$$

(a) $[-2, 0) \cap (0, 1)$ (b) $[-2, 0)$
(c) $[-2, 1)$ (d) $[-2, 0) \cup (0, 1)$

56. The trigonometric function $y = \tan x$ in the II quadrant

(a) decreases from 0 to ∞
(b) increases from 0 to ∞
(c) decreases from – ∞ to 0
(d) increases from – ∞ to 0

57. The degree measure of $\frac{\pi}{32}$ is equal to

(a) $5^\circ\,30'\,20''$ (b) $5^\circ\,37'\,30''$
(c) $5^\circ\,37'\,20''$ (d) $4^\circ\,30'\,30''$

58. The value of $\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$ is

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{4}$

59. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} =$

(a) $\sin 2\theta$ (b) $2\sin\theta$ (c) $2\cos\theta$ (d) $2\cos\frac{\theta}{2}$

60. If $A = \{1, 2, 3, ... 10\}$ then number of subsets of A containing only odd numbers is

(a) 31 (b) 32 (c) 27 (d) 30

SOLUTIONS

1. (None of the options is correct) :
Number of words starting with $A = 3! = 6$
Number of words starting with $K = 3! = 6$
Number of words starting with $M = 3! = 6$
So, 19th word starts from S, *i.e.*, SAKM

2. (c) : Let r be the common ratio of the given G.P.

$$\therefore \quad \frac{a_3}{a_1} = 25 \Rightarrow r^2 = 25 \Rightarrow r = \pm 5$$

$$\text{Now, } \frac{a_9}{a_5}=\frac{a_1r^8}{a_1r^4}=r^4=(\pm 5)^4=5^4$$

3. (c) : Since, the line passes through (7, 17) and (15, β) is perpendicular to the line $2x - 3y + 17 = 0$.

∴ Product of their slopes = – 1

$$\Rightarrow \left(\frac{\beta-17}{15-7}\right)\left(\frac{2}{3}\right)=-1 \qquad \Rightarrow \beta-17=\frac{-3\times 8}{2}=-12$$

$$\Rightarrow \beta = 5$$

4. (a)

5. (c) : $\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2}(x^2-1)=4-1=3$

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2}(2x+3)=4+3=7$$

∴ Quadratic equation whose roots are 3 and 7 is, $x^2 - 10x + 21 = 0$

6. (b): We have, $3x + i(4x - y) = 6 - i$

On comparing real and imaginary parts, we get

$3x = 6$ and $4x - y = -1$

$\Rightarrow x = 2$ and $4(2) - y = -1 \Rightarrow x = 2$ and $y = 9$

7. (d): We know that,

$$\text{S.D.} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$\Rightarrow 5 = \left\{\frac{(-1)^2 + 0^2 + (1)^2 + k^2}{4}\right\} - \left(\frac{-1+0+1+k}{4}\right)^2$$

$$\Rightarrow 5 = \frac{k^2+2}{4} - \frac{k^2}{16} \Rightarrow 80 = 4k^2 + 8 - k^2 \Rightarrow k^2 = 24$$

$$\Rightarrow k = 2\sqrt{6} \qquad (\because k > 0)$$

8. (a): $\because n(x) \neq n(y)$

So, there can't be any bijections.

9. (b): $f(-1) + f(2) + f(4)$

$= 3(-1) + (2)^2 + 2(4) = -3 + 4 + 8 = 9$

10. (a)

11. (a): We have, $\lim\limits_{y \to 0} \dfrac{\sqrt{3+y^3} - \sqrt{3}}{y^3}$

$$= \lim_{y \to 0}\left[\frac{\sqrt{3+y^3} - \sqrt{3}}{y^3} \times \frac{\sqrt{3+y^3} + \sqrt{3}}{\sqrt{3+y^3} + \sqrt{3}}\right]$$

$$= \lim_{y \to 0} \frac{(3+y^3-3)}{y^3\left(\sqrt{3+y^3}+\sqrt{3}\right)} = \lim_{y \to 0}\left(\frac{1}{\sqrt{3+y^3}+\sqrt{3}}\right) = \frac{1}{2\sqrt{3}}$$

12. (c): $(A^2)^{-1} = (A^{-1})^2$

13. (a): We have, $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\Rightarrow A^2 = I \Rightarrow A \cdot A^2 = A \cdot I \Rightarrow A^3 = A \Rightarrow (A^3)^{-1} = A^{-1}$

Now, $\text{adj } A = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$

and $|A| = -4 + 3 = -1 \neq 0$.

$$\therefore A^{-1} = (A^3)^{-1} = -1\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} = A$$

14. (d): We have, $A' = -A$ $\quad(\because A$ is skew- symmetric)

Now, $(A^{2021})' = (A')^{2021} = (-A)^{2021} = -1(A^{2021})$

$\therefore A^{2021}$ is also skew symmetric matrix.

15. (c): We have, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now, $(aI + bA)^n = {}^nC_0(aI)^n + {}^nC_1(aI)^{n-1}\, bA + {}^nC_2(aI)^{n-2}\,(bA)^2 + \ldots$

$= a^nI + n\,a^{n-1}\,I\,bA + 0 \qquad [\because A^2 = 0]$

$= a^nI + na^{n-1}\,bA$

16. (b): $|5 \cdot \text{adj } A| = 5 \Rightarrow 5^3 \cdot |\text{adj } A| = 5$

$$\Rightarrow |A|^{3-1} = \frac{1}{5^2} \Rightarrow |A|^2 = 5^{-2} \Rightarrow |A| = \pm\frac{1}{5}$$

17. (a): We have $\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

$\Rightarrow 1(2a^2 + 4) - 2(-4a - 20) = 86$

$\Rightarrow 2a^2 + 8a - 42 = 0 \Rightarrow a^2 + 4a - 21 = 0$

$\Rightarrow (a + 7)(a - 3) = 0 \Rightarrow a = -7$ or $a = 3$

18. (c): Area of triangle

$$= \frac{1}{2}\left|-2(-6-5) + 3(5-6) + 1(6+6)\right|$$

$$= \frac{1}{2}\left|22 - 3 + 12\right| = \frac{1}{2} \times 31 = 15.5 \text{ sq. units}$$

19. (d): We know that,

$0 \le \cos^{-1}[x] \le \pi \Rightarrow -1 \le [x] \le 1$

$\Rightarrow -1 \le x < 2 \Rightarrow x \in [-1, 2)$

20. (a): We have, $y = (1 + x^2)\tan^{-1}x - x$

$$\Rightarrow \frac{dy}{dx} = (1+x^2)\cdot\left(\frac{1}{1+x^2}\right) + \tan^{-1}(x)(2x) - 1$$

$$\Rightarrow \frac{dy}{dx} = 2x\tan^{-1}x$$

21. (a): We have, $x = e^{\theta}\sin\theta$ $\qquad$ ...(i)

$y = e^{\theta}\cos\theta$ $\qquad$ ...(ii)

Differentiating (i) & (ii) w.r.t. θ, we have

$$\frac{dx}{d\theta} = e^{\theta}\sin\theta + \cos\theta \cdot e^{\theta} = e^{\theta}(\sin\theta + \cos\theta)$$

and $\dfrac{dy}{d\theta} = e^{\theta}\cos\theta - \sin\theta \cdot e^{\theta} = e^{\theta}(\cos\theta - \sin\theta)$

Now, $\dfrac{dy}{dx} = \left(\dfrac{dy}{d\theta}\right)\Big/\left(\dfrac{dx}{d\theta}\right) = \dfrac{e^{\theta}(\cos\theta - \sin\theta)}{e^{\theta}(\cos\theta + \sin\theta)}$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - \tan\theta}{1 + \tan\theta} \qquad \text{...(iii)}$$

From (i) & (ii), we get $\frac{x}{y} = \tan\theta$

At (1, 1), we have, $\tan\theta = 1$

$\therefore$ From (iii), we get $\frac{dy}{dx} = 0$

22. (a): We have, $y = e^{\sqrt{x\sqrt{x\sqrt{x\ldots}}}}$, $x > 1$

$\Rightarrow \quad y = e^{x^{1/2} \cdot x^{1/4} \cdot x^{1/8} \ldots}$

$\Rightarrow \quad y = e^{x^{\left\{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\right\}}}$

$\Rightarrow y = e^{x^1}$ $\quad \left[\because \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots = \frac{1/2}{1-1/2} = 1\right]$

$\Rightarrow y = e^x \Rightarrow \frac{dy}{dx} = e^x$

$\Rightarrow \frac{d^2y}{dx^2} = e^x \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=\log_e 3} = e^{\log_e 3} = 3$

23. (c): Let $y = f(f(f(x))) + (f(x))^2$

$\Rightarrow y' = f'(f(f(x))\, f'(f(x)) \cdot f'(x) + 2f(x) \cdot f'(x)$

$\therefore \quad y' \text{ (at } x = 1) = f'\,(f(f(1))) \cdot f'(f(1)) \cdot f'(1) + 2f(1) \cdot f'(1)$

$= f'(f(1)) f'(1) \cdot f'(1) + 2 \cdot 1 \cdot 3$

$= f'(1) \cdot f'(1) \cdot f'(1) + 6 = 27 + 6 = 33$

24. (b): We have, $y = x^{\sin x} + (\sin x)^x$

$y = u + v$, where $u = x^{\sin x}$, $v = (\sin x)^x$

$\Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ...(i)

Now, $u = x^{\sin x} \Rightarrow \log u = \sin x \log x$

Differentiate w.r.t. x, we get

$\frac{1}{u}\frac{du}{dx} = \cos x \cdot \log x + \frac{1}{x}\sin x$

$\Rightarrow \quad \frac{du}{dx} = x^{\sin x}\left\{\frac{x \log x \cdot \cos x + \sin x}{x}\right\}$ ...(ii)

Similarly, $\log v = x \cdot \log \sin x$

$\Rightarrow \quad \frac{1}{v}\frac{dv}{dx} = x \cdot \frac{1}{\sin x} \cdot \cos x + \log \sin x$

$\Rightarrow \quad \frac{dv}{dx} = (\sin x)^x \left\{\frac{x \cos x + \sin x \cdot \log \sin x}{\sin x}\right\}$ ...(iii)

From (i), (ii), & (iii), we get

$$\frac{dy}{dx} = x^{\sin x}\left\{\frac{x \log x \cdot \cos x + \sin x}{x}\right\} + (\sin x)^x \left\{\frac{x \cos x + \sin x \cdot \log \sin x}{\sin x}\right\}$$

$\therefore \quad \left.\frac{dy}{dx}\right|_{x=\pi/2} = \left(\frac{\pi}{2}\right)^1 \left\{\frac{0+1}{\pi/2}\right\} + (1)^{\pi/2}\left\{\frac{0+0}{1}\right\} = 1$

25. (c): We have, $A_n = \begin{bmatrix} 1-n & n \\ n & 1-n \end{bmatrix}$

$\Rightarrow \quad |A_n| = \begin{vmatrix} 1-n & n \\ n & 1-n \end{vmatrix} = (1-n)^2 - n^2$

$\Rightarrow \quad |A_n| = 1 - 2n$

$\therefore \quad |A_1| + |A_2| + \ldots + |A_{2021}|$

$= (1 - 2 \cdot 1) + (1 - 2 \cdot 2) + (1 - 2 \cdot 3) + \ldots + (1 - 2 \cdot 2021)$

$= [1 + 1 + \ldots \text{ (2021 times)}] - 2\,[1 + 2 + 3 \ldots + 2021]$

$= 2021 - 2 \cdot \left(\frac{2021 \cdot 2022}{2}\right)$

$= 2021\,(1 - 2022) = -\,(2021)^2$

26. (b): We have, $f(x) = \log(1 + x) - \frac{2x}{2+x}$

$\Rightarrow \quad f'(x) = \frac{1}{1+x} - \left\{\frac{(2+x) \cdot 2 - 2x \cdot 1}{(2+x)^2}\right\}$

$= \frac{(2+x)^2 - 4(1+x)}{(1+x)(2+x)^2} = \left(\frac{1}{1+x}\right) \cdot \left(\frac{x}{2+x}\right)^2$

For increasing, $f'(x) \geq 0$

$\Rightarrow \quad 1 + x > 0 \Rightarrow x > -1$ $\quad \left[\because \left(\frac{x}{2+x}\right)^2 \geq 0\right]$

$\therefore \quad f(x)$ is increasing, if $x \in (-1, \infty)$

27. (b): Let the required point be $p(a, b)$

Given equation of curve is,

$\sqrt{x} + \sqrt{y} = 6$ ...(i)

$\because \quad p$ lies on the curve (i). So, p satisfies it

$\Rightarrow \quad \sqrt{a} + \sqrt{b} = 6$ ...(ii)

Differentiating (i) w.r.t. x, we get

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\sqrt{\frac{y}{x}}$

But tangent is equally inclined to the axes.

$\therefore \quad$ Slope = tan 45° = 1 $\Rightarrow \frac{dy}{dx} = 1$

$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{(a,b)} = 1 \Rightarrow -\sqrt{b} = \sqrt{a} \Rightarrow b = a$

So, from (ii), we have

$2\sqrt{a} = 6 \Rightarrow \sqrt{a} = 3 \Rightarrow a = 9$

$\therefore \quad$ Required point $p = (9, 9)$

28. (d): We have, $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$

$\Rightarrow f'(x) = 12\sin^2 x \cdot \cos x - 12\sin x \cdot \cos x + 12\cos x$

$= 12\cos x(\sin^2 x - \sin x + 1) = 12\cos x\{\sin^2 x + (1 - \sin x)\}$

Now, $\sin^2 x > 0$ and $(1 - \sin x) \geq 0$

$\therefore\ f'(x) > 0$ if $\cos x > 0$ or $f'(x) < 0$ if $\cos x < 0$

$$\Rightarrow\ x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \text{ or } x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

So, $f(x)$ is strictly increasing in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

29. (a) : Let $I = \int_0^8 [x]\,dx$

$$= \int_0^1 [x]\,dx + \int_1^2 [x]\,dx + \ldots + \int_7^8 [x]dx$$

$$= \int_0^1 0\cdot dx + \int_1^2 1\cdot dx + \int_2^3 2\cdot dx + \int_3^4 3\cdot dx + \int_4^5 4\cdot dx + \int_5^6 5\cdot dx + \int_6^7 6\cdot dx + \int_7^8 7\cdot dx$$

$$= 0 + [x]_1^2 + [2x]_2^3 + [3x]_3^4 + [4x]_4^5 + [5x]_5^6 + [6x]_6^7 + [7x]_7^8$$

$= (2 - 1) + 2(3 - 2) + 3(4 - 3) + 4(5 - 4) + 5(6 - 5) + 6(7 - 6) + 7(8 - 7)$

$$= \frac{7\cdot 8}{2} = 28$$

30. (b): Let $I = \int_0^{\pi/2} \sqrt{\sin\theta}\cdot\cos^3\theta\,d\theta$

Put $\sin\theta = t \Rightarrow \cos\theta\,d\theta = dt$

When $\theta = 0$, $t = \sin 0 = 0$

And when $\theta = \frac{\pi}{2}, t = \sin\frac{\pi}{2} = 1$

$$\therefore\ I = \int_0^1 (t)^{1/2}\cdot(1 - \sin^2\theta)\,dt$$

$$= \int_0^1 (t^{1/2})(1 - t^2)\,dt = \int_0^1 (t^{1/2} - t^{5/2})dt$$

$$= \left[\frac{2}{3}t^{3/2} - \frac{2}{7}t^{7/2}\right]_0^1 = \frac{2}{3} - \frac{2}{7} = \frac{8}{21}$$

31. (d): We have, $e^y + xy = e$ ...(i)

Differentiating w.r.t. x, we get

$$e^y\cdot\frac{dy}{dx} + x\frac{dy}{dx} + y = 0 \qquad \text{...(ii)}$$

$$\Rightarrow\ \frac{dy}{dx} = \left(\frac{-y}{e^y + x}\right) \Rightarrow \left.\frac{dy}{dx}\right|_{x=0} = \frac{-1}{e}$$

$[\because$ At $x = 0$, from (i) we have $e^y + 0 = e \Rightarrow y = 1]$

Again, differentiating (ii) w.r.t. x, we get

$$e^y\cdot\left(\frac{dy}{dx}\right)^2 + e^y\cdot\frac{d^2y}{dx^2} + x\frac{d^2y}{dx^2} + \frac{2dy}{dx} = 0$$

$$\Rightarrow\ \frac{d^2y}{dx^2} = -\left[\frac{e^y\left(\frac{dy}{dx}\right)^2 + 2\left(\frac{dy}{dx}\right)}{e^y + x}\right]$$

$$\Rightarrow\ \left.\frac{d^2y}{dx^2}\right|_{x=0} = -\left[\frac{e^1\left(\frac{-1}{e}\right)^2 + 2\left(\frac{-1}{e}\right)}{e}\right] = \frac{1}{e^2}$$

$\therefore$ Required ordered pair $= \left(\frac{-1}{e}, \frac{1}{e^2}\right)$

32. (c) : Let $I = \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}dx$

$$= \int \frac{(2\cos^2 x - 1) - (2\cos^2\alpha - 1)}{\cos x - \cos\alpha}dx$$

$$= \int \frac{2(\cos x - \cos\alpha)(\cos x + \cos\alpha)}{\cos x - \cos\alpha}dx$$

$$= 2\int \cos x\,dx + 2\int \cos\alpha\,dx = 2(\sin x + x\cos\alpha) + c$$

33. (d): Let $I = \int_0^1 \frac{x\,e^x}{(2+x)^3}dx = \int_0^1 \frac{(x+2-2)\,e^x}{(2+x)^3}dx$

$$= \int_0^1 e^x\left[\frac{(x+2)}{(x+2)^3} - \frac{2}{(x+2)^3}\right]dx$$

$$= \int_0^1 e^x\left[\frac{1}{(x+2)^2} - \frac{2}{(x+2)^3}\right]dx = \left[e^x\cdot\frac{1}{(x+2)^2}\right]_0^1$$

$$\left[\because\ \int e^x(f(x) + f'(x))dx = e^x\cdot f(x) + c\right]$$

$$= \frac{e}{9} - \frac{1}{4}$$

34. (a) : Let $I = \int \frac{dx}{(x+2)(x^2+1)}$ ...(i)

By partial fractions,

Let $\frac{1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx + C}{x^2+1}$

$\Rightarrow\ 1 = A(x^2 + 1) + (x + 2)(Bx + C)$

$\Rightarrow\ 1 = (A + B)x^2 + (2B + C)x + (A + 2C)$

On comparing the coefficients of x^2, x and constant term, we get

$A + B = 0,\ 2B + C = 0,\ A + 2C = 1$

On solving above equations, we get

$$A = \frac{1}{5}, B = \frac{-1}{5}, C = \frac{2}{5}$$

$\therefore$ (i) becomes,

$$I=\int\frac{1}{5}\left(\frac{dx}{x+2}\right)+\frac{1}{5}\int\left(\frac{2-x}{x^2+1}\right)dx$$

$$=\frac{1}{5}\log|x+2|+\frac{2}{5}\int\frac{1}{x^2+1}dx-\frac{1}{5\cdot 2}\int\frac{2x}{x^2+1}dx+c$$

$$=\frac{1}{5}\log|x+2|+\frac{2}{5}\tan^{-1}x-\frac{1}{10}\log|x^2+1|+c$$

$$=a\log|1+x^2|+b\tan^{-1}x+\frac{1}{5}\log|x+2|+c$$

$$\Rightarrow\quad a=\frac{-1}{10}\text{ and }b=\frac{2}{5}$$

35. (c) : Required area = $\int_0^{\pi/3}\tan x\,dx$

$$=\left|\left[-\log\cos x\right]_0^{\pi/3}\right|=\left|\left(-\log\frac{1}{2}+\log 1\right)\right|=\log 2$$

36. (d): Here, $a = 2, b = 3, f(x) = x^2, nh = b - a = 1$

$$\therefore\quad \int_2^3 x^2\,dx=\lim_{h\to 0}\Big[h\big\{2^2+(2+h)^2+(2+2h)^2+\ldots\big\} +(2+(n-1)h)^2\Big]$$

$$=\lim_{h\to 0}\Big[h\big\{(2^2+2^2+\ldots\text{upto }n\text{ times})+h^2\{1^2+2^2 +\ldots+(n-1)^2\}+4h\,(1+2+3+\ldots(n-1))\big\}\Big]$$

$$=\lim_{h\to 0}\left[h\left\{4n+h^2\cdot\left(\frac{(n-1)\cdot n\cdot(2n-1)}{6}\right)+4h\cdot\frac{(n-1)n}{2}\right\}\right]$$

$$=\lim_{h\to 0}\left[nh(4)+\frac{(nh-h)\cdot nh(2n\,h-h)}{6}+2(nh-h)\cdot nh\right]$$

$$=4+\frac{1}{3}+2=\frac{19}{3}$$

37. (d): Let $I=\int_0^{\pi/2}\frac{\cos x\cdot\sin x}{1+\sin x}dx$

Put $1 + \sin x = t \Rightarrow \cos x\,dx = dt$

When $x = 0$, $t = 1$ and when $x = \pi/2$, $t = 2$

$$\therefore\quad I=\int_1^2\frac{(t-1)}{t}dt=\int_1^2\left(1-\frac{1}{t}\right)dt$$

$$\Rightarrow\quad I=\left[t-\log|t|\right]_1^2=2-\log 2-1+0$$

$\Rightarrow\quad I = 1 - \log 2$

38. (c) : The given differential equation is,

$\frac{dy}{dx}+\frac{y}{x}=x^2$, which is linear differential equation.

$$\therefore\quad \text{I.F.}=e^{\int\frac{1}{x}dx}=e^{\log x}=x$$

So, general solution is ,

$$x\cdot y=\int x^3\,dx+C$$

$$\Rightarrow\quad xy=\frac{x^4}{4}+C\Rightarrow y=\frac{x^3}{4}+C(x^{-1})$$

$$\therefore\quad 2y(2)-y(1)=2\left[\frac{8}{4}+\frac{C}{2}\right]-\left[\frac{1}{4}+C\right]=\frac{15}{4}$$

39. (a) : The given differential equation is,

$$\frac{dy}{dx}=(x+y)^2 \qquad \text{...(i)}$$

Put $x + y = t \Rightarrow 1+\frac{dy}{dx}=\frac{dt}{dx}$

So, (i) becomes,

$$\Rightarrow\quad \frac{dt}{dx}-1=t^2\Rightarrow\frac{dt}{dx}=t^2+1$$

$$\Rightarrow\quad \int\frac{dt}{t^2+1}=\int dx+c\Rightarrow\tan^{-1}t=x+c$$

$$\Rightarrow\quad \tan^{-1}(x+y)=x+c$$

40. (b): The given differential equation is,

$$x\log x\cdot\frac{dy}{dx}+y=2x\log x\quad\Rightarrow\quad\frac{dy}{dx}+\frac{1}{x\log x}\cdot y=2$$

$$\therefore\quad \text{I.F.}=e^{\int\frac{1}{x\log x}dx}=e^{\log(\log x)}=\log x$$

So, general solution is, $y\cdot\log x=2\int\log x\,dx+C$

$\Rightarrow\quad y\log x = 2[x\log x - x] + C$

At $x = 1$, we have $C = 2$

$$\Rightarrow\quad y=2x-\frac{2x}{\log x}+\frac{2}{\log x}$$

$\therefore\quad y(e) = 2e - 2e + 2 = 2$

41. (c) : $\left|\frac{1}{2}\vec{a}-\frac{1}{3}\vec{b}\right|^2=\frac{1}{4}|\vec{a}|^2+\frac{1}{9}|\vec{b}|^2-2\cdot\frac{1}{2}\cdot\frac{1}{3}(\vec{a}\cdot\vec{b})$

$$=\frac{1}{4}\times 4+\frac{1}{9}\times 9-\frac{1}{3}|\vec{a}||\vec{b}|\cos\theta$$

$$=2-\frac{1}{3}\times 2\times 3\cdot\cos(120^\circ)\quad=2+2\cdot\frac{1}{2}=3$$

42. (The question is incorrect) : Correct question should be $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=36$

$$\Rightarrow\quad |\vec{a}|^2|\vec{b}|^2(\sin^2\theta+\cos^2\theta)=36$$

$$\Rightarrow\quad 9\cdot|\vec{b}|^2=36\Rightarrow|\vec{b}|^2=4\Rightarrow|\vec{b}|=2$$

Then, answer should be 2 *i.e.*, option (d).

43. (None of the options is correct) :

We have, $\vec{\alpha} = \hat{i} - 3\hat{j}$, $\vec{\beta} = \hat{i} + 2\hat{j} - \hat{k}$

$\because\ \vec{\beta}_1$ is parallel to $\vec{\alpha}$

$\therefore\ \vec{\beta}_1 = \lambda\,(\hat{i} - 3\hat{j})$

Also, $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$

$\Rightarrow\ \vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1 = \hat{i} + 2\hat{j} - \hat{k} - \lambda(\hat{i} - 3\hat{j})$

$\Rightarrow\ \vec{\beta}_2 = (1-\lambda)\hat{i} + (2+3\lambda)\,\hat{j} - \hat{k}$

It is given that β_2 is perpendicular to $\vec{\alpha}$.

$\therefore\ \vec{\alpha}\cdot\vec{\beta}_2 = 0$

$\Rightarrow\ (\hat{i} - 3\hat{j})\cdot\left((1-\lambda)\hat{i} + (2+3\lambda)\hat{j} - \hat{k}\right) = 0$

$\Rightarrow\ (1 - \lambda) - 3\,(2 + 3\lambda) = 0$

$\Rightarrow\ 1 - \lambda - 6 - 9\,\lambda = 0 \Rightarrow -5 = 10\lambda \Rightarrow \lambda = \dfrac{-1}{2}$

So, $\vec{\beta}_1 = \dfrac{-1}{2}\left(\hat{i} - 3\hat{j}\right)$

44. (b): The given differential equation is,

$\left(1 + y_1^2\right)^{2/3} = y_2 \Rightarrow \left(1 + y_1^2\right)^2 = y_2^3$

So, order = 2, degree = 3

$\therefore$ Required sum = 5

45. (b): Let $P(x_1, y_1, z_1)$ be the foot of perpendicular drawn from the point $O(0, 0, 0)$ to the plane

$2x - 3y + 4z = 29$.

Dr's of $OP = \langle x_1, y_1, z_1 \rangle$

Dr's of normal to the given plane = $\langle 2, -3, 4 \rangle$

$\therefore$ Equation of line OP is,

$$\frac{x_1}{2} = \frac{y_1}{-3} = \frac{z_1}{4} = \lambda \text{ (say)}$$

$\Rightarrow\ x_1 = 2\lambda,\ y_1 = -3\lambda,\ z_1 = 4\lambda$

$\because\ P(x_1, y_1, z_1)$ lies on plane.

$\therefore\ 2(2\lambda) + 3(3\lambda) + 4\,(4\lambda) = 29$

$\Rightarrow\ 4\lambda + 9\lambda + 16\lambda = 29 \Rightarrow \lambda = 1$

$\therefore$ Required coordinates of foot of perpendicular are $(2, -3, 4)$.

46. (None of the options is correct)

$$\cos\theta = \left|\frac{3\cdot 1 + 5\cdot 4 + 4\cdot 2}{\sqrt{3^2 + 5^2 + 4^2}\cdot\sqrt{1^2 + 4^2 + 2^2}}\right| = \left|\frac{3 + 20 + 8}{5\sqrt{2}\cdot\sqrt{21}}\right|$$

$$\Rightarrow\ \theta = \cos^{-1}\left(\frac{31}{5\sqrt{42}}\right)$$

47. (d): We have

Corner points	Value of $z = 4x + 6y$
(0, 2)	$4 \times 0 + 6 \times 2 = 12$
(3, 0)	$4 \times 3 + 6 \times 0 = 12$
(6, 0)	$4 \times 6 + 6 \times 0 = 24$
(6, 8)	$4 \times 6 + 6 \times 8 = 72$
(0, 5)	$4 \times 0 + 6 \times 5 = 30$

So, minimum of z occurs at only two points.

48. (a) : Let the dietician uses x packets of food X and y packets of food Y.

So, the mathematical formulation of the given problems is,

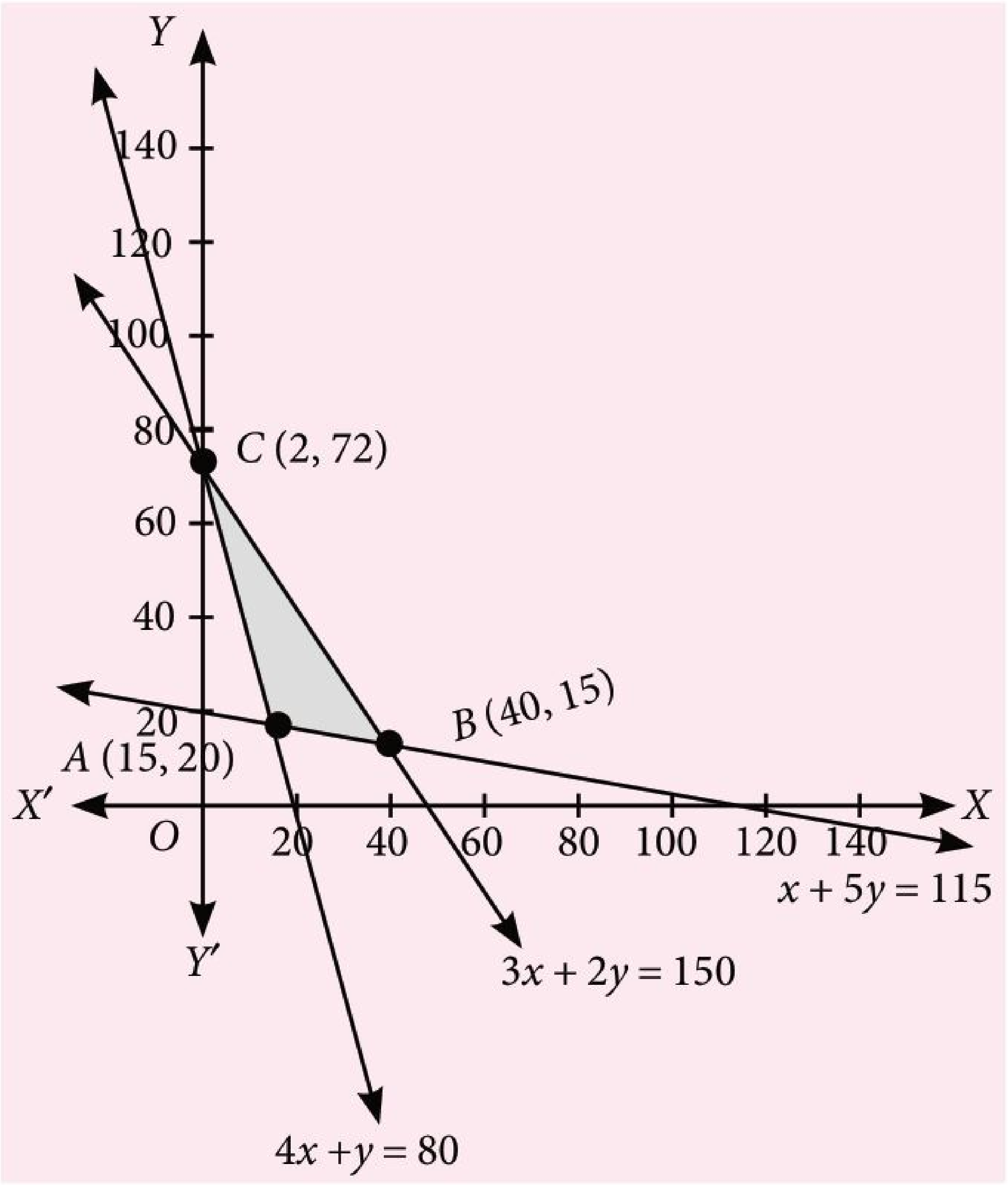
$12x + 3y \geq 240 \Rightarrow 4x + y \geq 80$ ...(i)

$4x + 20y \geq 460 \Rightarrow x + 5y \geq 115$ ...(ii)

$6x + 4y \leq 300 \Rightarrow 3x + 2y \leq 150$ ...(iii)

$x \geq 0, y \geq 0$

The graph of the above equation is as follows :



The shaded region is the feasible region.

So, the required corner points are $A(15, 20)$, $B(40, 15)$, and $C(2, 72)$.

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49. (a) : The given point is $(2\hat{i}+\hat{j}-\hat{k})$

So, required distance $=\dfrac{|(2\hat{i}+\hat{j}-\hat{k})\cdot(\hat{i}-2\hat{j}+4\hat{k})-4|}{\sqrt{1^2+2^2+4^2}}$

$=\dfrac{|(2-2-4-4)|}{\sqrt{21}}=\dfrac{8}{\sqrt{21}}$

50. (a) : The sample space associated with the given experiment is

$S=\{$HHT, HTT, HTH, HHH, TTH, TTT, THH, THT$\}$

Let X be the random variable indicating the number of heads.

So, X can take values 0, 1, 2, 3.

The corresponding probabilities are,

$P(x=0)=P(\text{no head})=\dfrac{1}{8}$

$P(x=1)=P(\text{one head})=\dfrac{3}{8}$

$P(x=2)=P(2\text{ heads})=\dfrac{3}{8}$

$P(x=3)=P(3\text{ heads})=\dfrac{1}{8}$

Now, mean $=\displaystyle\sum_{X=0}^{3}X\cdot P(X)=0+\frac{3}{8}+\frac{6}{8}+\frac{3}{8}=\frac{12}{8}=1.5$

51. (a) : $P(A|B)=\dfrac{1}{4}\Rightarrow\dfrac{P(A\cap B)}{P(B)}=\dfrac{1}{4}$

$P(A\cap B)=\dfrac{1}{4}\times\dfrac{1}{3}=\dfrac{1}{12}$

Now, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{12}=\dfrac{3}{4}$

$\therefore\ P(A'\cap B')=P(A\cup B)'=1-P(A\cup B)=1-\dfrac{3}{4}=\dfrac{1}{4}$

52. (a) : Let E_1 be the event that there is a lockdown and E_2 be the event that there is no lockdown.

Let A be the event that pandemic is controlled in one month.

$\therefore\ P(E_1)=0.7,\ P(E_2)=0.3$

$P(A/E_1)=0.8,\ P(A/E_2)=0.3$

So, $P(A)=P(A/E_1)\cdot P(E_1)+P(A/E_2)\cdot P(E_2)$

$=(0.8)(0.7)+(0.3)(0.3)=0.65$

53. (c) : We have, $P(\bar{A})=0.75$

$\Rightarrow\ P(A)=1-0.75=0.25$

$\because$ A and B are independent events.

$\therefore\ P(A\cap B)=P(A)\,P(B)$

Now, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow\ 0.65=0.25+x-0.25x\Rightarrow 0.4=0.75x\Rightarrow x=\dfrac{8}{15}$

54. (a) : We have, $n(A)=p$, $n(B)=q$

$n(A\times B)=p\times q\Rightarrow 7=pq$

$\Rightarrow$ Either $p=1$, $q=7$ or $q=1$, $p=7$

$\therefore\ p^2+q^2=1^2+7^2=50$

55. (d): For domain, $x+2\geq 0\Rightarrow x\geq -2$ ...(i)

$1-x>0\Rightarrow x<1$ ...(ii)

Also, $\log_{10}(1-x)\neq 0$

$\Rightarrow\ 1-x\geq 1\Rightarrow x\neq 0$ ...(iii)

From (i), (ii), & (iii), we get

$x\in[-2,0)\cup(0,1)$

56. (d)

57. (b): $\dfrac{\pi}{32}=\dfrac{\pi}{32}\times\dfrac{180^\circ}{\pi}=\left(\dfrac{180}{32}\right)^\circ=5^\circ\left(\dfrac{20}{32}\right)^\circ$

$=5^\circ\left(\dfrac{20\times 60}{32}\right)'$

$=5^\circ 37'\left(\dfrac{16}{32}\right)^\circ=5^\circ 37'\left(\dfrac{16\times 60}{32}\right)''=5^\circ 37' 30''$

58. (d): We have, $\sin\dfrac{5\pi}{12}\cdot\sin\dfrac{\pi}{12}=\sin\left(\dfrac{\pi}{2}-\dfrac{\pi}{12}\right)\cdot\sin\dfrac{\pi}{12}$

$=\cos\dfrac{\pi}{12}\cdot\sin\dfrac{\pi}{12}=\dfrac{1}{2}\cdot\left(2\sin\dfrac{\pi}{12}\cdot\cos\dfrac{\pi}{12}\right)$

$=\dfrac{1}{2}\cdot\sin\dfrac{2\pi}{12}=\dfrac{1}{2}\cdot\sin\dfrac{\pi}{6}=\dfrac{1}{4}$

59. (c) : We have,

$\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=\sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8\theta)}}}$

$=\sqrt{2+\sqrt{2+\sqrt{2\cdot 2\cos^2 4\theta}}}$

$=\sqrt{2+\sqrt{2+2\cos 4\theta}}=\sqrt{2+\sqrt{2(1+\cos 4\theta)}}$

$=\sqrt{2+2\cdot\cos 2\theta}=\sqrt{2\cdot(1+\cos 2\theta)}=\sqrt{2\cdot 2\cos^2\theta}=2\cos\theta$

60. (b): Number of odd numbers in the given set $A=5$

$\therefore$ Required number of subsets $=2^5=32$

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CLASS-XI

Series-1 Sets | Relations and Functions

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections*-I *and* II.
2. *Section-I comprises of* 16 *very short answer type questions.*
3. *Section-II contains* 2 *case study-based questions.*

Part - B :

1. *It consists of three Sections*-III, IV *and* V.
2. *Section*-III *comprises of* 10 *questions of* 2 *marks each.*
3. *Section*-IV *comprises of* 7 *questions of* 3 *marks each.*
4. *Section*-V *comprises of* 3 *questions of* 5 *marks each.*
5. *Internal choice is provided in 3 questions of Section*-III, *2 questions of Section*-IV *and 3 questions of Section*-V. *You have to attempt only one of the alternatives in all such questions.*

PART - A

SECTION - I

1. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 2^x$, then find range of f.

2. If $A = \{1, 3, 5, 6\}$. Find the number of elements in $P\{P(A)\}$.

OR

If $A \subseteq B$, then find the value of $B' - A'$.

3. If $f(x) = x + 1$ and $g(x) = 2x - 3$ be two real functions, find the value of $f^2 - 3g$.

4. If $a \in N$ such that $aN = \{ax : x \in N\}$. Describe the set $3N \cap 7N$.

OR

Let $A = \{(x, y) : y = e^{2x}, \forall\, x \in R\}$ and $B = \{(x, y) : y = e^{-2x}, \forall\, x \in R\}$, then find $A \cap B$.

5. Find domain of the function $f(x) = \dfrac{1}{\sqrt{|x| - x}}$.

6. Find the domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

OR

Find the range of the function $f(x) = \dfrac{|x-4|}{x-4}$.

7. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 4, 6\}$. We define xRy if $x < y$. Is R a relation from A to B?

8. Describe the set $\{x : x \in Z \text{ and } |x| \le 2\}$ in roster form.

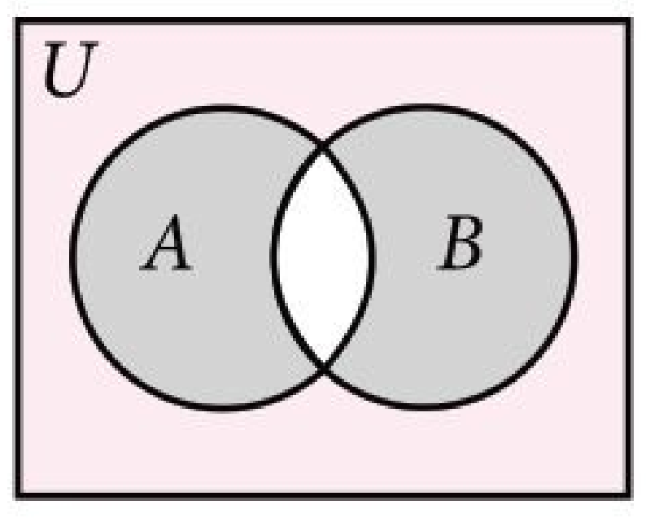
OR

If A and B are two sets, then prove that $(A \cup B')' \cap (A' \cup B)'$ is a null set.

9. If $A = \{1, 2, 3, 4\}$ and $x, y \in A$, then form the set of all ordered pairs (x, y) such that x is a divisor of y.

10. If $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 3, 5, 6, 7, 8, 9\}$, then show that $A - B \ne B - A$.

11. Find the expression of shaded region for the given figure.



12. If $f(x) = e^x$, then find $\dfrac{f(a)}{f(-b)}$.

OR

Let $n(A) = 6$ and $n(B) = p$. Then, find the total number of non-empty relations that can be defined from A to B.

13. If $f(x) = 9x - x^2$, $x \in R$, then find $f(a + 1) - f(a - 1)$.

14. Let $A = \{x : x^2 - 5x + 6 = 0\}$, $B = \{2, 4\}$, $C = \{4, 5\}$, then find $A \times (B \cap C)$.

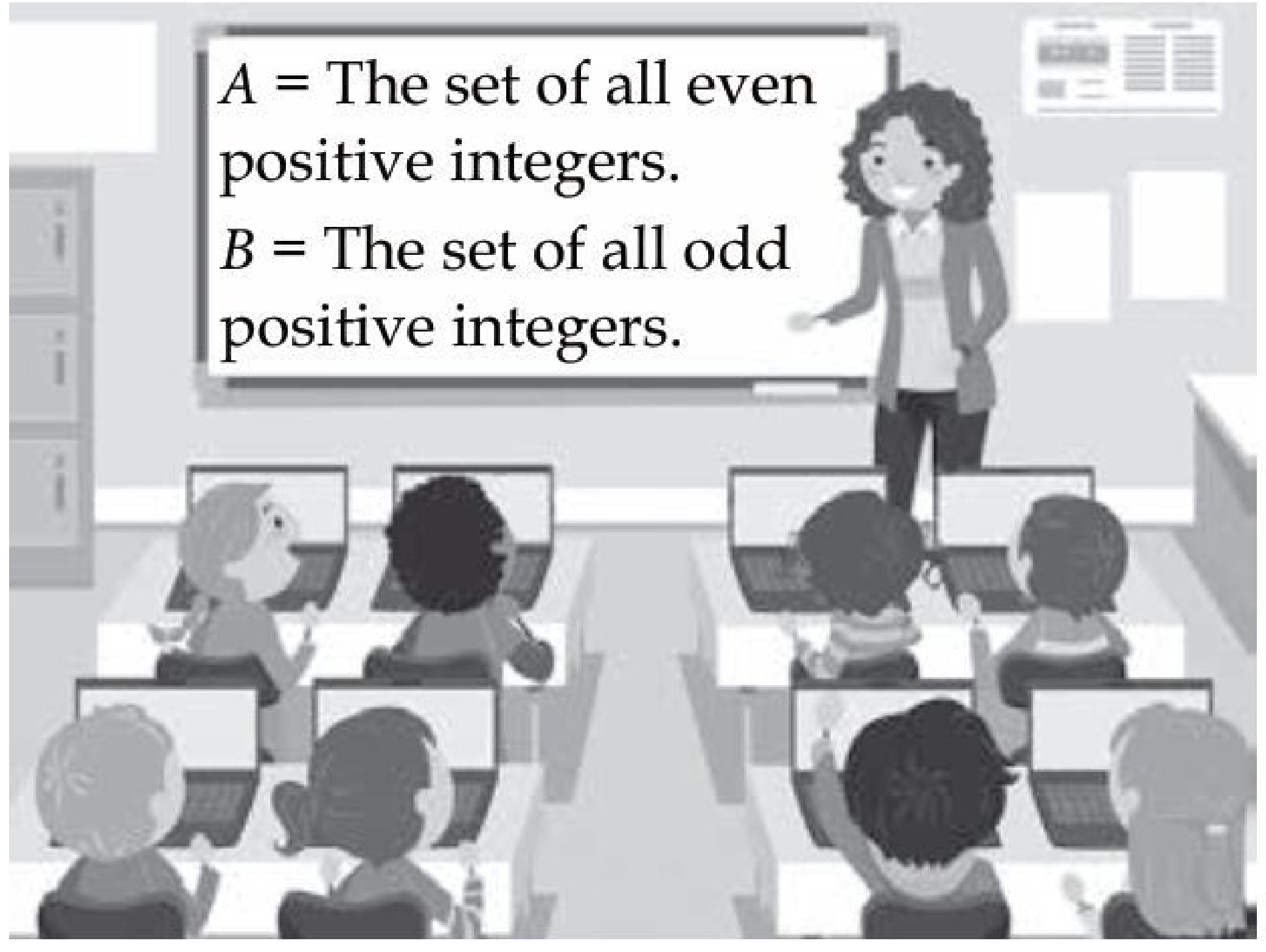
15. Find the range of function $f(x) = \dfrac{x}{|x|}$.

16. Let $A = \{3, 5, 6\}$ and $B = \{2, 7\}$. Represent $A \times B$
(i) graphically (ii) by arrow diagram

SECTION - II

Case study based questions are compulsory. Attempt any 4 sub parts from each question. Each sub part carries 1 mark.

17. Daksh is studying in class XI. In evening, he went to his tuition class, where the tuition teacher wrote the sets A and B on the board and asked some questions based on it.

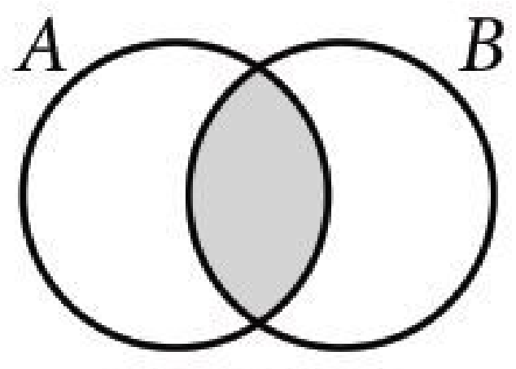


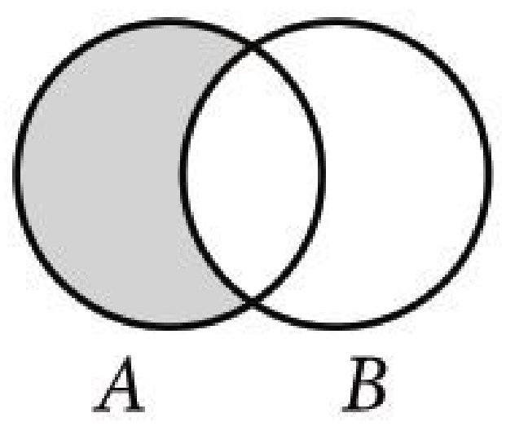
A = The set of all even positive integers.
B = The set of all odd positive integers.

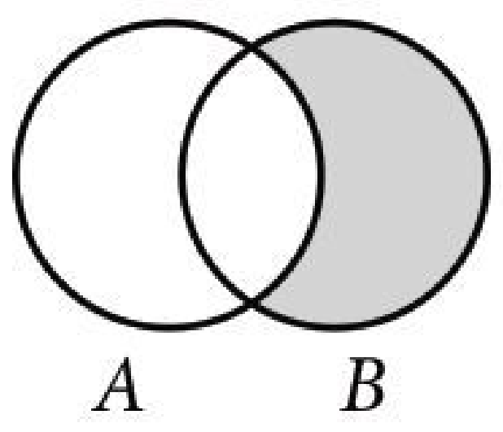
Based on the above information, answer the following questions.

(i) The set $A \cup B$ is
(a) N (b) Z (c) R (d) Q

(ii) Venn-diagram of $(A \cap B) \cap (A \cup B)$ is

(a)


(b)


(c)


(d) None of these

(iii) The set $(A - B) \cup (B - A)$ is
(a) $A \cap B$ (b) $A \cup B$
(c) $A - B$ (d) $B - A$

(iv) $(A \cup B)' =$
(a) N' (b) Z' (c) Q' (d) ϕ

(v) Find $n(A \cap B)$.
(a) 0 (b) 1
(c) 2 (d) None of these

18. Ram made a cubical box by folding metal sheet, the side of cubical box is x cm. Let y be the volume of the box.

Based on the given information, answer the following questions.

(i) Which of the following function represents the given situation?
(a) $y = 3x$ (b) $y = x^3$
(c) $x = y^3$ (d) None of these.

(ii) Domain of the given function is
(a) $(-\infty, -1) \cup (1, \infty)$ (b) $(0, \infty)$
(c) $(-1, 1)$ (d) None of these.

(iii) Range of the given function is
(a) $(-\infty, 0)$
(b) set of real positive numbers
(c) set of natural numbers
(d) none of these.

(iv) If $y = x^5$, then y is
(a) even function (b) odd function
(c) $y = f(5) = 3025$ (d) none of these.

(v) If $f(x) = x^3$, then the value of $f(1) \times f(3)$ is
(a) 27 (b) 81 (c) 729 (d) 3

PART-B

SECTION - III

19. State which of the following sets are finite or infinite.
(i) $A = \{x : x \in N \text{ and } x \text{ is odd}\}$
(ii) $B = \{x : x$ is a month of a year having less than 31 days$\}$.

20. If $f(x) = \dfrac{|x|}{x}$, $x \neq 0$, then prove that $|f(\alpha) - f(-\alpha)| = 2$, $\alpha \neq 0$.

21. If A and B are two non-empty sets having n elements in common, then prove that $A \times B$ and $B \times A$ have n^2 elements in common.

22. If $A = \{0\}$, $B = \{x : x < 5 \text{ and } x > 15\}$,
$C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$,
$E = \{x : x$ is a positive integral root of the equation $x^2 - 2x - 15 = 0\}$, then find all pairs of equal sets (if any).

23. Find $f(x) + f(1 - x)$, if $f(x) = \dfrac{a^x}{a^x + \sqrt{a}}$.

OR

Let f be defined by $f(x) = x - 4$ and g be defined by

$$g(x) = \begin{cases} \dfrac{x^2 - 16}{x + 4}, & x \neq -4 \\ \lambda, & x = -4 \end{cases}.$$

Find λ such that $f(x) = g(x)$ for all $x \in \mathbb{R}$.

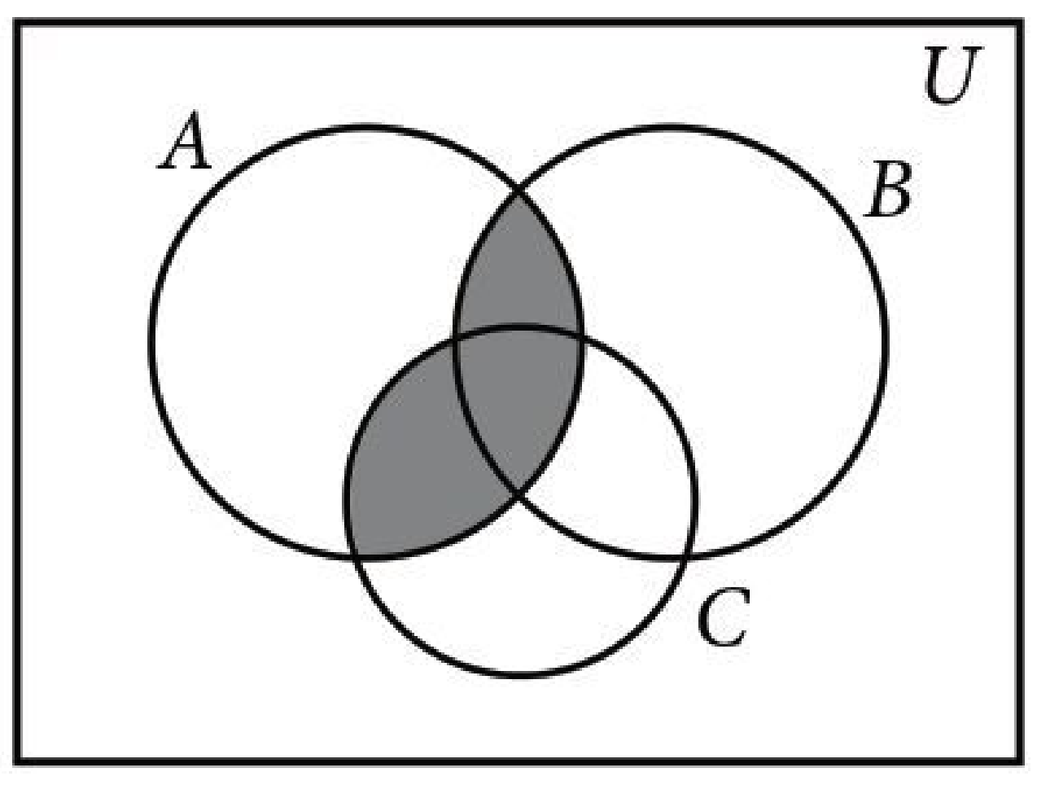
24. If $A \cap B' = \phi$, then show that $A \subseteq B$.

25. If $A = \{-1, 1\}$, then form the set $A \times A \times A$.

OR

If $R = \{(x, y) : x, y \in Z, x^2 + y^2 = 64\}$, then write R in roster form.

26. What is represented by the shaded region in the following Venn diagram?



27. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7, 8\}$, then verify that $A - B = A \cap B'$.

28. If A and B are two sets such that $n(A \times B) = 6$ and some elements of $A \times B$ are $(-1, 2)$, $(2, 3)$, $(4, 3)$, then find $A \times B$ and $B \times A$.

OR

Consider the relation $f(x) = \begin{cases} x^2, & 0 \leq x \leq 5 \\ 4x, & 5 \leq x \leq 10 \end{cases}$.

Show that f is not a function.

SECTION - IV

29. Find the domain and the range of the relation R given by $R = \{(x, y) : y = x + \dfrac{6}{x}$, where $x, y \in N$ and $x < 6\}$.

30. Let $f = \left\{\left(x, \dfrac{x^2}{1 + x^2}\right), x \in \mathbb{R}\right\}$ be a function from $\mathbb{R}$ into $\mathbb{R}$. Determine the range of f.

OR

Let $A = \{-3, -2, -1, 4\}$ and $f : A \to Z$ given by $f(x) = x^2 + x + 2$. Find
(a) the range of f
(b) pre-images of 6 and 4.

31. If $A = \{1, 3, 5, \ldots, 17\}$ and $B = \{2, 4, 6, \ldots, 18\}$ and N = the set of natural numbers is the universal set, then show that $A' \cup ((A \cup B) \cap B') = N$.

32. Let R be a relation from Q to Q defined by $R = \{(a, b) : a, b \in Q \text{ and } a - b \in Z\}$. Show that
(a) $(a, a) \in R$ for all $a \in Q$.
(b) $(a, b) \in R$ implies that $(b, a) \in R$
(c) $(a, b) \in R$ and $(b, c) \in R$ implies that $(a, c) \in R$

33. Two finite sets A and B have m and k elements respectively. If the ratio of total number of subsets of A to total number of subsets of B is 64 : 1 and $n(A) + n(B) = 12$, then find the values of m and k.

OR

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$,
$B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$.
Verify : $A - (B \cup C)' = (A \cap B) \cup (A \cap C)$

34. If $f(x) = \dfrac{1 + x}{1 - x}$, prove that $\dfrac{f(x) \cdot f(x^2)}{1 + [f(x)]^2} = \dfrac{1}{2}$.

35. Find the union of each of the following pairs of sets.

(i) $A = \{a, e, i, o, u\}$, $B = \{a, c, d\}$

(ii) $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$

(iii) $A = \{x : x$ is a natural number and $1 < x \le 5\}$ and $B = \{x : x$ is a natural number and $5 < x \le 10\}$

SECTION - V

36. If $f(x) = e^x$ and $g(x) = \log x$, then find the following functions:

(i) $f+g$ (ii) $f-g$ (iii) fg (iv) $\dfrac{f}{g}$

(v) $\dfrac{1}{f}$ (vi) f^2.

OR

Find the domain of the function $f(x)$ given by $f(x) = \log_4\{\log_5(\log_3(18x - x^2 - 77))\}$.

37. Let A be the set of first ten natural numbers and let R be a relation on A defined by $(x, y) \in A \Leftrightarrow x + 2y = 10$, *i.e.*, $R = \{(x, y) : x \in A, y \in A$ and $x + 2y = 10\}$. Express R as sets of ordered pairs. Also, determine

(i) domain of R (ii) range of R.

OR

Find the domain and range of the following real valued functions :

(a) $f(x) = \dfrac{1}{\sqrt{x^2 - 81}}$ (b) $f(x) = \dfrac{2x}{6 - x}$

38. If $U = \{x : x \in N, x \le 30\}$, $A = \{x : x$ is prime $< 5\}$, $B = \{x : x$ is a perfect square $\le 10\}$ and $C = \{x : x$ is a perfect cube $\le 30\}$, then show that

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

(iii) $(A \cap B) \cap C = A \cap (B \cap C)$

OR

Verify : (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Using Venn diagram.

SOLUTIONS

1. Given $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 2^x$.

Since $D_f = \mathbb{R}$ and for all $x \in \mathbb{R}$, $2^x > 0$

$\therefore$ Range of $f = (0, \infty)$.

2. Given, $A = \{1, 3, 5, 6\}$

$\therefore$ $n\{P(A)\} = 2^4 = 16 \Rightarrow n[P\{P(A)\}] = 2^{16}$

OR

Let $x \in B' \Rightarrow x \notin B \Rightarrow x \notin A \Rightarrow x \in A'$

$\Rightarrow$ $B' \subseteq A' \Rightarrow B' - A' = \phi$.

3. $(f^2 - 3g)(x) = (f^2)(x) - (3g)(x) = (f(x))^2 - 3g(x)$

$= (x + 1)^2 - 3(2x - 3) = x^2 + 2x + 1 - 6x + 9$

$= x^2 - 4x + 10$, for all $x \in R$.

4. We have, $aN = \{ax : x \in N\}$

$\therefore$ $3N = \{3x : x \in N\} = \{3, 6, 9, 12, \ldots\}$

and $7N = \{7x : x \in N\} = \{7, 14, 21, 28, \ldots\}$

Hence, $3N \cap 7N = \{21, 42, \ldots\} = \{21x : x \in N\} = 21N$.

OR

Given $A = \{(x, y) : y = e^{2x}, \forall x \in R\}$

and $B = \{(x, y) : y = e^{-2x}, \forall x \in R\}$

$\Rightarrow$ $e^{2x} = \dfrac{1}{e^{2x}}$ $\Rightarrow e^{4x} = 1 \Rightarrow e^{4x} = e^0$

$\Rightarrow$ $x = 0$ and $y = 1$

As e^{2x} and e^{-2x} both intersect each other at (0, 1).

$\therefore$ $A \cap B = \{(0, 1)\}$ *i.e.*, a singleton set.

5. We have, $f(x) = \dfrac{1}{\sqrt{|x| - x}}$

Now, $|x| - x > 0 \Rightarrow |x| > x$

Thus, x must be negative $\Rightarrow x \in (-\infty, 0)$.

6. The values of x for which $f(x)$ and $g(x)$ are equal are given by $f(x) = g(x)$

$\Rightarrow$ $2x^2 - 1 = 1 - 3x \Rightarrow 2x^2 + 3x - 2 = 0$

$\Rightarrow$ $(x + 2)(2x - 1) = 0 \Rightarrow x = -2, 1/2$.

Thus, $f(x)$ and $g(x)$ are equal on the set $\{-2, 1/2\}$.

OR

We have, $f(x) = \dfrac{|x - 4|}{x - 4}$

Clearly, $f(x)$ is defined for all $x \in \mathbb{R}$ except at $x = 4$.

$$\text{Now, } f(x) = \begin{cases} \dfrac{x-4}{x-4} = 1 & , \text{ if } x > 4 \\ \dfrac{-(x-4)}{x-4} = -1 & , \text{ if } x < 4 \end{cases}$$

Hence, Range$(f) = \{-1, 1\}$.

7. We have, xRy if $x < y$

$\therefore$ $R = \{(1, 2), (1, 4), (1, 6), (2, 4), (2, 6), (3, 4), (3, 6), (4, 6), (5, 6)\}$

Here, $R \subseteq A \times B$ $\therefore$ R is a relation from A to B.

8. Here x is an integer satisfying $|x| \le 2$.

$\Rightarrow$ $|x| = 0, 1, 2 \Rightarrow x = 0, \pm 1, \pm 2$

So, x can take values $-2, -1, 0, 1, 2$.

$\therefore$ $\{x : x \in Z$ and $|x| \le 2\} = \{-2, -1, 0, 1, 2\}$.

OR

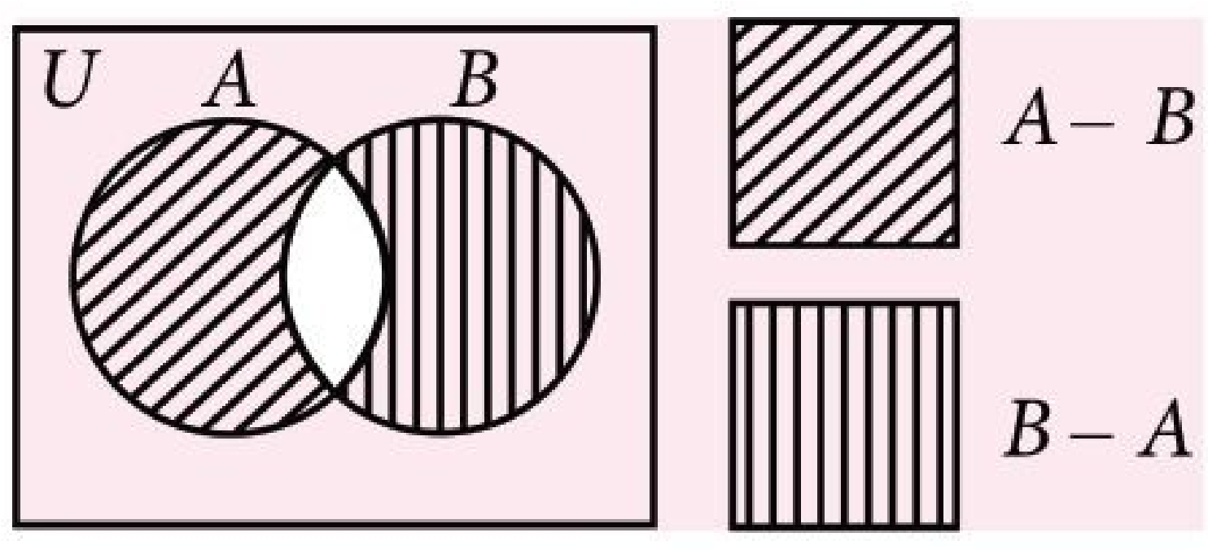
Consider $(A \cup B')' \cap (A' \cup B)'$

$= (A' \cap B) \cap (A \cap B') = (B - A) \cap (A - B) = \phi$

9. Given $A = \{1, 2, 3, 4\}$ and $x, y \in A$.
The set of all ordered pairs (x, y) such that x is a divisor of $y = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)\}$.

10. Here $A - B = \{2, 4\}$ and $B - A = \{9\}$
$\therefore$ $A - B \neq B - A$.

11.



$\therefore$ Shaded part represents $(A - B) \cup (B - A)$.

12. $\dfrac{f(a)}{f(-b)} = \dfrac{e^a}{e^{-b}} = e^a e^b = e^{a+b} = f(a+b)$

OR

Given, $n(A) = 6$ and $n(B) = p$
$\therefore$ Total number of relations from A to $B = 2^{6p}$
$\therefore$ Total number of non-empty relations from A to $B = 2^{6p} - 1$

13. $f(a + 1) - f(a - 1)$
$= [9(a + 1) - (a + 1)^2] - [9(a - 1) - (a - 1)^2]$
$= 18 - 4a = 2(9 - 2a)$.

14. $A = \{x : x^2 - 5x + 6 = 0\}$
$x^2 - 5x + 6 = 0 \Rightarrow (x - 3)(x - 2) = 0 \Rightarrow x = 3, 2$
$\therefore$ $A = \{2, 3\}, B = \{2, 4\}, C = \{4, 5\}$
Now, $B \cap C = \{4\}$
$\Rightarrow$ $A \times (B \cap C) = \{2, 3\} \times \{4\} = \{(2, 4), (3, 4)\}$.

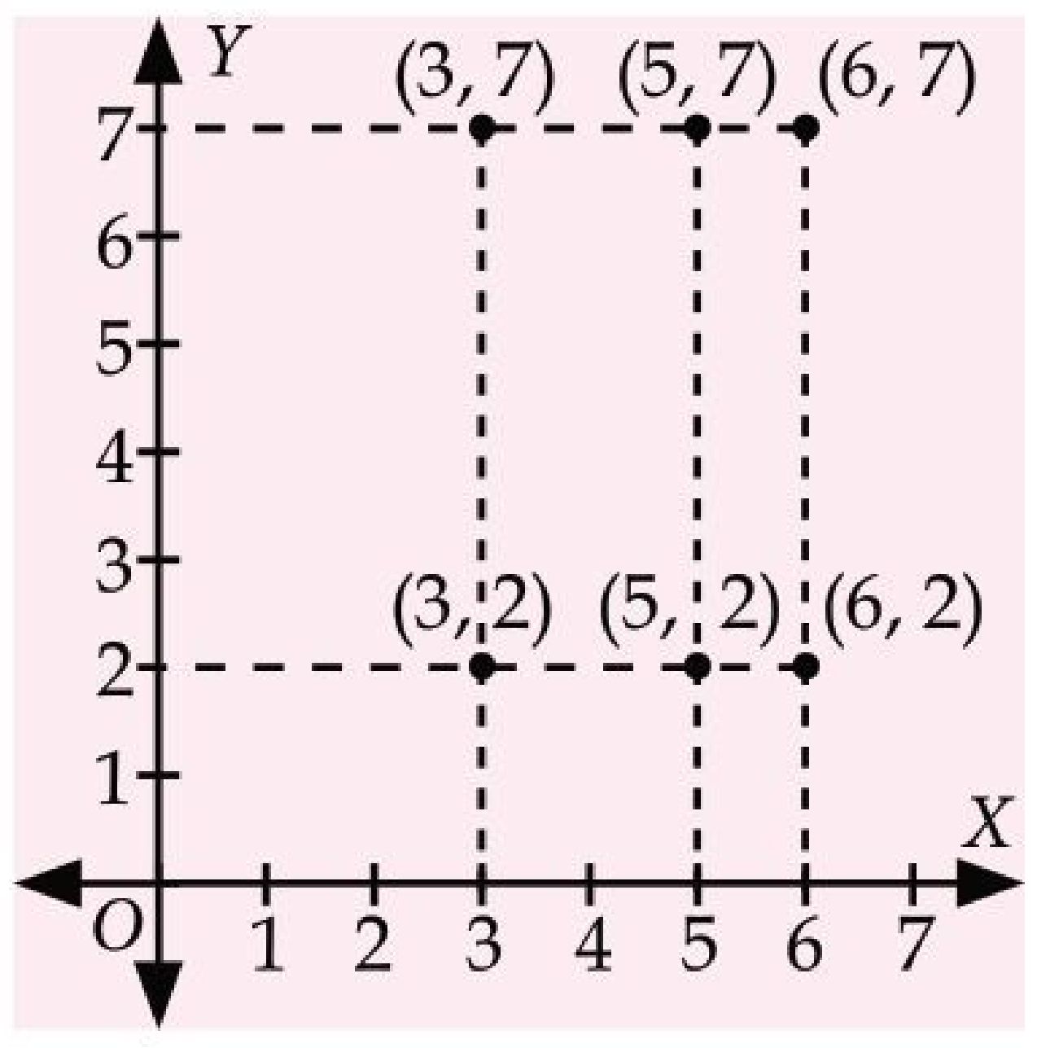
15. Here, $f(x) = \dfrac{x}{|x|}$. Cleraly, $f(x)$ is defined for all $x \in R$ except at $x = 0$.

Now, $f(x) = \begin{cases} 1 & , \text{if } x > 0 \\ -1 & , \text{if } x < 0 \end{cases}$

$\therefore$ Range $(f) = \{-1, 1\}$

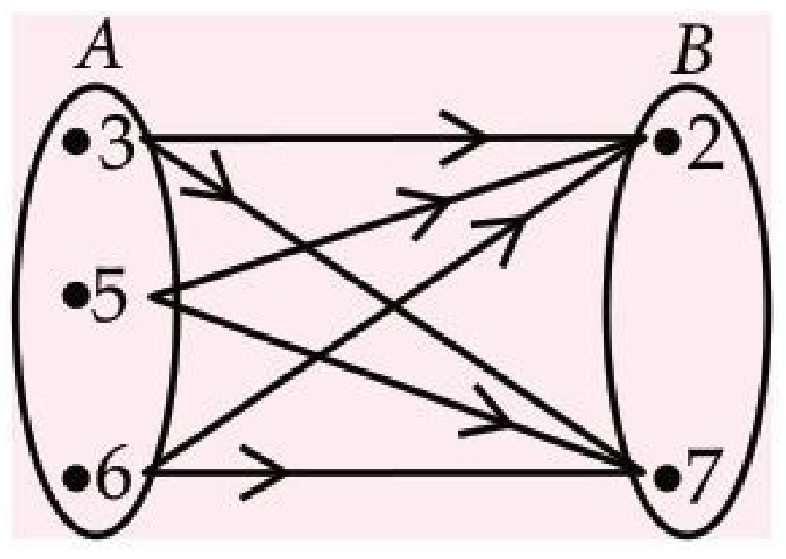
16. Here $A = \{3, 5, 6\}$ and $B = \{2, 7\}$

(i) Graphical representation of $A \times B$ is shown by dots in the given figure:



$\therefore$ $A \times B = \{(3, 2), (3, 7), (5, 2), (5, 7), (6, 2), (6, 7)\}$

(ii) Arrow diagram of $A \times B$ is :



17. **(i) (a) :** As, $A \cup B = N$

(ii) (d) : As, $A \cap B = \phi$ and $A \cup B = N$
$\therefore$ $(A \cap B) \cap (A \cup B) = \phi \cap N = \phi$

(iii) (b) : As, $A - B = A - (A \cap B)$
$= A - \phi = A$ $\quad [\because A \cap B = \phi]$
Also, $B - A = B - (B \cap A) = B - \phi = B$
$\therefore$ $(A - B) \cup (B - A) = A \cup B$

(iv) (a) : Since, $A \cup B = N \Rightarrow (A \cup B)' = N'$

(v) (a) : As, $A \cap B = \phi \Rightarrow n(A \cap B) = 0$

18. **(i) (b) :** $y = x^3$ represents the given situation.

(ii) (b) : The side of a cube is always positive, *i.e.*, $x > 0$
$\therefore$ Domain $= (0, \infty)$

(ii) (b) : Range of the function $y = x^3$ is the set of all positive real numbers.

(iv) (b) : $y = x^5$ is an odd function.

(v) (a) : We have, $f(x) = x^3$
$f(1) = 1$ and $f(3) = 27$
$\therefore$ $f(1) \times f(3) = 27$

19. (i) $A = \{x : x \in N$ and x is odd$\} = \{1, 3, 5, 7, 9, 11, \ldots\}$.
Since odd numbers are infinite in numbers, so A is an infinite set.
(ii) $B = \{x : x$ is a month of a year having less than 31 days$\}$, *i.e.*, {February, April, June, September, November}.
$\therefore$ B is a finite set.

20. $f(\alpha) - f(-\alpha) = \dfrac{|\alpha|}{\alpha} - \dfrac{|-\alpha|}{-\alpha} = \dfrac{|\alpha|}{\alpha} + \dfrac{|-\alpha|}{\alpha}$

$= \dfrac{|\alpha|}{\alpha} + \dfrac{|\alpha|}{\alpha} = 2\dfrac{|\alpha|}{\alpha} = \begin{cases} 2, & \alpha > 0 \\ -2, & \alpha < 0 \end{cases}$

Thus, $f(\alpha) - f(-\alpha) = \pm 2$
$\therefore$ $|f(\alpha) - f(-\alpha)| = |\pm 2| = 2$.

21. Let $C = A \cap B$.
$\therefore$ $C \times C = (A \cap B) \times (A \cap B) = (A \cap B) \times (B \cap A)$
$= (A \times B) \cap (B \times A)$
Since $A \cap B$ has n elements, so C has n elements.
Hence $C \times C$ has n^2 elements.
$\therefore$ $(A \times B) \cap (B \times A)$ have n^2 elements.
Hence $A \times B$ and $B \times A$ have n^2 elements in common.

22. The given sets are:
$A = \{0\}, B = \phi, C = \{5\}, D = \{5, -5\}, E = \{5\}$
$(\because\ x^2 - 2x - 15 = 0 \Rightarrow (x-5)(x+3) = 0 \Rightarrow x = 5, -3)$
Thus, the only pair of equal sets is C and E.

23. Given, $f(x) = \dfrac{a^x}{a^x + \sqrt{a}}$ ...(i)

$$\therefore\quad f(1-x) = \frac{a^{1-x}}{a^{1-x} + \sqrt{a}} = \frac{a \cdot a^{-x}}{a \cdot a^{-x} + \sqrt{a}}$$

$$= \frac{a}{a + \sqrt{a} \cdot a^x} = \frac{\sqrt{a}}{\sqrt{a} + a^x} \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$f(x) + f(1-x) = \frac{a^x}{a^x + \sqrt{a}} + \frac{\sqrt{a}}{\sqrt{a} + a^x} = \frac{a^x + \sqrt{a}}{a^x + \sqrt{a}} = 1$$

OR

We have, $f(x) = g(x)$ for all $x \in R$
$\Rightarrow\ f(-4) = g(-4) \ \Rightarrow\ -4 - 4 = \lambda$
$\Rightarrow\ \lambda = -8.$

24. Given, $A \cap B' = \phi$, *i.e.*, $A - B = \phi$
Let $x \in A$ be an arbitrary element.
Now, if $x \notin B$, then $x \in A - B$
But $A - B = \phi$
Thus, $x \notin B$ is not possible.
$\Rightarrow$ x will definitely belong to B
Hence, $A \subseteq B$. [$\because\ x \in A$ was arbitrary]

25. Here $A = \{-1, 1\}$
$\therefore\ A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}.$

OR

Here $R = \{(x, y) : x, y \in Z, x^2 + y^2 = 64\}.$
When $x = 0, y = 8, -8;$
When $x = 8, y = 0$; when $x = -8, y = 0.$
For all other values of $x \in Z$, we do not get $y \in Z$.
$\therefore\ R = \{(0, 8), (0, -8), (8, 0), (-8, 0)\}.$

26. The shaded area represents
$(A \cap B) \cup (A \cap C) = A \cap (B \cup C).$

27. Clearly, $A - B = \{1, 2, 4, 6, 8\} - \{1, 3, 5, 7, 8\}$
$= \{2, 4, 6\}$ ...(i)
and $B' = U - B = \{2, 4, 6, 9\}$
Now, $A \cap B' = \{1, 2, 4, 6, 8\} \cap \{2, 4, 6, 9\}$
$= \{2, 4, 6\}$ ...(ii)
From (i) and (ii), it is clear that $A - B = A \cap B'$.

28. It is given that $n(A \times B) = 6 = 3 \times 2$
$= n(A) \times n(B)$
Also, $(-1, 2), (2, 3), (4, 3) \in A \times B$.
So, $A = \{-1, 2, 4\}$ and $B = \{2, 3\}$.
$\therefore\ A \times B = \{(-1, 2), (-1, 3), (2, 2), (2, 3), (4, 2), (4, 3)\}$
and $B \times A = \{(2, -1), (3, -1), (2, 2), (3, 2), (2, 4), (3, 4)\}.$

OR

$$\text{As, } f(x) = \begin{cases} 25\ ; & x = 5 \\ 20\ ; & x = 5 \end{cases}$$

Since, f has two different images of f at $x = 5$.
$\therefore$ f is not a function.

29. Given $y = x + \dfrac{6}{x}$, $x, y \in N$ and $x < 6$.

When $x = 1, y = 1 + \dfrac{6}{1} = 7$ and $7 \in N$, so $(1, 7) \in R$,

When $x = 2, y = 2 + \dfrac{6}{2} = 5$ and $5 \in N$, so $(2, 5) \in R$,

When $x = 3, y = 3 + \dfrac{6}{3} = 5$ and $5 \in N$, so $(3, 5) \in R$,

When $x = 4, y = 4 + \dfrac{6}{4} \notin N$, and

When $x = 5, y = 5 + \dfrac{6}{5} \notin N$

$\Rightarrow\ R = \{(1, 7), (2, 5), (3, 5)\}.$
$\therefore$ Domain of $R = \{1, 2, 3\}$ and range of $R = \{5, 7\}$.

30. Let $f = \left\{\left(x, \dfrac{x^2}{x^2+1}\right), x \in \mathbb{R}\right\}$

We know that $f(x) = \dfrac{x^2}{x^2+1}$ is defined for all $x \in \mathbb{R}$

as $x^2 + 1 \neq 0$ for any $x \in \mathbb{R}$.
So, domain of $f(x)$ is real number.
Let $f(x) = y$

$$\Rightarrow\ \frac{x^2}{x^2+1} = y \ \Rightarrow\ x = \pm\sqrt{\frac{y}{1-y}}$$

$\therefore$ x will be defined when $\dfrac{y}{1-y} \geq 0$
i.e., $0 \leq y < 1 \Rightarrow y \in [0, 1)$
Hence, range of $f = [0, 1)$

OR

(a) We have, $f(x) = x^2 + x + 2$
$\Rightarrow\ f(-3) = (-3)^2 + (-3) + 2 = 8,$
$f(-2) = (-2)^2 + (-2) + 2 = 4,$
$f(-1) = (-1)^2 + (-1) + 2 = 2$
and $f(4) = (4)^2 + 4 + 2 = 22$
Therefore, range of $f = \{8, 4, 2, 22\}$
(b) Let x be the pre-image of 6.
Then, $f(x) = 6 \ \Rightarrow\ x^2 + x + 2 = 6$

$$\Rightarrow\ x^2 + x - 4 = 0 \ \Rightarrow\ x = \frac{-1 \pm \sqrt{17}}{2}$$

Since $\frac{-1\pm\sqrt{17}}{2} \notin A,$ therefore, there is no pre-image of 6.

Now, for pre-image of 4

$x^2 + x + 2 = 4 \Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x + 2)(x - 1) = 0 \Rightarrow x = -2, 1$

Since $-2 \in A$, therefore, -2 is the pre-image of 4.

31. Given $A = \{1, 3, 5, \ldots, 17\}$ and $B = \{2, 4, 6, \ldots, 18\}$,

$\therefore \quad A \cap B = \phi \qquad \ldots\text{(i)}$

Now, $A' \cup ((A \cup B) \cap B') = A' \cup ((A \cap B') \cup (B \cap B'))$ (Distributive law)

$= A' \cup ((A \cap B') \cup \phi) \qquad (\because B \cap B' = \phi)$

$= A' \cup (A \cap B') \qquad (\because A \cup \phi = A)$

$= (A' \cup A) \cap (A' \cup B') \qquad$ (Distributive law)

$= N \cap (A \cap B)' \qquad$ (De-Morgan's law)

$= N \cap \phi' \qquad$ [Using (i)]

$= N \cap N = N$

32. We have, $R = \{(a, b) : a, b \in Q \text{ and } a - b \in Z\}$

(a) Let $a \in Q$. Then $a - a = 0 \in Z$.

$\therefore \quad (a, a) \in R \ \forall \ a \in Q$.

(b) $(a, b) \in R \Rightarrow (a - b) \in Z$

$\Rightarrow -(a - b) \in Z \ (\because (-1) \in Z) \Rightarrow b - a \in Z$

$\Rightarrow (b, a) \in R$

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$

(c) Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow a - b \in Z$ and $b - c \in Z \Rightarrow \{(a - b) + (b - c)\} \in Z$

$\Rightarrow (a - c) \in Z \Rightarrow (a, c) \in R$

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

33. Given, $n(A) = m$ and $n(B) = k$

$\Rightarrow n(P(A)) = 2^m$ and $n(P(B)) = 2^k$

Ratio of $n(P(A))$ to $n(P(B)) = 64 : 1$ [Given]

$\therefore \quad \frac{2^m}{2^k} = \frac{64}{1} \Rightarrow 2^{m-k} = 64$

$\Rightarrow 2^{m-k} = 2^6 \Rightarrow m - k = 6 \qquad \ldots\text{(i)}$

Also, $n(A) + n(B) = 12 \Rightarrow m + k = 12 \qquad \ldots\text{(ii)}$

Solving (i) and (ii), we get $m = 9$ and $k = 3$.

OR

We have, $B \cup C = \{2, 4, 6, 8\} \cup \{3, 4, 5, 6\}$

$= \{2, 3, 4, 5, 6, 8\} \qquad \ldots\text{(i)}$

$A \cap B = \{1, 2, 3, 4\} \cap \{2, 4, 6, 8\} = \{2, 4\} \qquad \ldots\text{(ii)}$

$A \cap C = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\} = \{3, 4\} \qquad \ldots\text{(iii)}$

L.H.S. $= A - (B \cup C)'$

$= \{1, 2, 3, 4\} - \{2, 3, 4, 5, 6, 8\}' \qquad$ (From (i))

$= \{1, 2, 3, 4\} - \{1, 7, 9\}$

$= \{2, 3, 4\} \qquad \ldots\text{(iv)}$

R.H.S. $= (A \cap B) \cup (A \cap C)$

$= \{2, 4\} \cup \{3, 4\} \qquad$ (From (ii) and (iii))

$= \{2, 3, 4\} \qquad \ldots\text{(v)}$

From (iv) and (v), we get L.H.S. = R.H.S.

34. Given, $f(x) = \frac{1+x}{1-x} \Rightarrow f(x^2) = \frac{1+x^2}{1-x^2}$

Now, $$\frac{f(x)\cdot f(x^2)}{1+[f(x)]^2} = \frac{\left(\frac{1+x}{1-x}\right)\left(\frac{1+x^2}{1-x^2}\right)}{1+\left(\frac{1+x}{1-x}\right)^2}$$

$$= \frac{\frac{1+x^2}{(1-x)^2}}{\frac{(1-x)^2+(1+x)^2}{(1-x)^2}} = \frac{1+x^2}{2(1+x^2)} = \frac{1}{2}.$$

35. (i) Given, $A = \{a, e, i, o, u\}$, $B = \{a, c, d\}$

$\Rightarrow A \cup B = \{a, c, d, e, i, o, u\}$

(ii) Given, $A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$

$\Rightarrow A \cup B = \{1, 2, 3, 4, 5, 6\}$

(iii) Given, $A = \{x : x \text{ is a natural number and } 1 < x \leq 5\}$

$\Rightarrow A = \{2, 3, 4, 5\}$

and $B = \{x : x \text{ is a natural number and } 5 < x \leq 10\}$

$\Rightarrow B = \{6, 7, 8, 9, 10\}$

Now, $A \cup B = \{2, 3, 4, 5\} \cup \{6, 7, 8, 9, 10\}$

$= \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

36. Given $f(x) = e^x$ and $g(x) = \log x$

$D_f = \mathbb{R}$ and $D_g = \mathbb{R}^+$, where $\mathbb{R}^+$ is the set of positive reals.

Let $D = D_f \cap D_g = \mathbb{R} \cap \mathbb{R}^+ = \mathbb{R}^+$

(i) $(f + g)(x) = f(x) + g(x) = e^x + \log x$, with domain $\mathbb{R}^+$.

(ii) $(f - g)(x) = f(x) - g(x) = e^x - \log x$, with domain $\mathbb{R}^+$.

(iii) $(fg)(x) = f(x)\, g(x) = e^x \log x$, with domain $\mathbb{R}^+$.

(iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{e^x}{\log x}$, with domain D_1,

where $D_1 = \mathbb{R}^+ - \{1\}$.

(v) $\left(\frac{1}{f}\right)(x) = \frac{1}{f(x)} = \frac{1}{e^x}$, with domain $\mathbb{R}$.

(vi) $(f^2)(x) = (f(x))^2 = (e^x)^2 = e^{2x}$, with domain $D_f = \mathbb{R}$.

OR

We have, $f(x) = \log_4\{\log_5(\log_3(18x - x^2 - 77))\}$

Since $\log_a x$ is defined for all $x > 0$.

$\therefore \quad f(x)$ is defined if

$\log_5\{\log_3(18x - x^2 - 77)\} > 0$ and $18x - x^2 - 77 > 0$

$\Rightarrow \log_3(18x - x^2 - 77) > 5^0$ and $x^2 - 18x + 77 < 0$

$\Rightarrow \log_3(18x - x^2 - 77) > 1$ and $(x - 11)(x - 7) < 0$

$\Rightarrow 18x - x^2 - 77 > 3^1$ and $7 < x < 11$

$\Rightarrow 18x - x^2 - 80 > 0$ and $7 < x < 11$

$\Rightarrow x^2 - 18x + 80 < 0$ and $7 < x < 11$

$\Rightarrow (x - 10)(x - 8) < 0$ and $7 < x < 11$

$\Rightarrow 8 < x < 10$ and $7 < x < 11 \Rightarrow 8 < x < 10$

$\Rightarrow x \in (8, 10)$.

Hence, the domain of $f(x)$ is $(8, 10)$.

37. We have, $(x, y) \in R$

$\Leftrightarrow \quad x + 2y = 10 \Leftrightarrow y = \dfrac{10-x}{2}, x, y \in A,$

where $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Now, $x = 1 \Rightarrow y = \dfrac{10-1}{2} = \dfrac{9}{2} \notin A$.

This shows that 1 is not related to any element in A. Similarly, 3, 5, 7, 9 and 10 are not related to any element of A under the defined relation.

For $x = 2, y = \dfrac{10-2}{2} = 4 \in A, \quad \therefore (2, 4) \in R$

For $x = 4, y = \dfrac{10-4}{2} = 3 \in A, \quad \therefore (4, 3) \in R$

For $x = 6, y = \dfrac{10-6}{2} = 2 \in A, \quad \therefore (6, 2) \in R$

For $x = 8, y = \dfrac{10-8}{2} = 1 \in A, \quad \therefore (8, 1) \in R$

$\therefore \quad R = \{(2,4), (4,3), (6,2), (8,1)\}$

(i) Domain = $\{2, 4, 6, 8\}$, (ii) Range = $\{1, 2, 3, 4\}$.

OR

(a) Here, $f(x) = \dfrac{1}{\sqrt{x^2-81}}$

$f(x)$ is defined for $x^2 - 81 > 0 \Rightarrow x^2 > 81 \Rightarrow x < -9, x > 9$

$\therefore$ Domain of f is $(-\infty, -9) \cup (9, \infty)$.

Let $y = f(x)$

$\Rightarrow \quad y = \dfrac{1}{\sqrt{x^2-81}} \Rightarrow y^2 = \dfrac{1}{x^2-81}$

$\Rightarrow \quad x^2 - 81 = \dfrac{1}{y^2} \Rightarrow x = \pm\sqrt{81 + \dfrac{1}{y^2}}$

$\therefore$ Range of $f = (0, \infty)$

(b) Here, $f(x) = \dfrac{2x}{6-x}$

$f(x)$ is not defined for $6 - x = 0 \Rightarrow x = 6$

$\therefore$ Domain of $f = R - \{6\}$

Let $y = f(x) \Rightarrow y = \dfrac{2x}{6-x} \Rightarrow 6y - xy = 2x$

$\Rightarrow \quad 6y = 2x + xy \Rightarrow x(2 + y) = 6y \Rightarrow x = \dfrac{6y}{2+y}$

$\Rightarrow$ Range of f is $R - \{-2\}$.

38. Given, $U = \{1, 2, 3, \ldots, 30\}, A = \{2, 3\}, B = \{1, 4, 9\}$ and $C = \{1, 8, 27\}$.

(i) $A \cup B = \{1, 2, 3, 4, 9\}$

$\Rightarrow \quad (A \cup B)' = \{5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$

Now, $A' = \{1, 4, 5, 6, \ldots, 30\}$ and

$B' = \{2, 3, 5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$

$\Rightarrow \quad A' \cap B' = \{5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$.

$\therefore \quad (A \cup B)' = A' \cap B'$

(ii) $A \cap B = \{2, 3\} \cap \{1, 4, 9\} = \phi \Rightarrow (A \cap B)' = U$

$A' \cup B' = \{1, 4, 5, 6, \ldots, 30\} \cup \{2, 3, 5, 6, 7, 8, 10, 11, 12, \ldots, 30\}$
$= \{1, 2, 3, 4, \ldots, 30\} = U \therefore (A \cap B)' = A' \cup B'$

(iii) $A \cap B = \{2, 3\} \cap \{1, 4, 9\} = \phi$

$\Rightarrow \quad (A \cap B) \cap C = \phi \cap C = \phi \cap \{1, 8, 27\} = \phi$

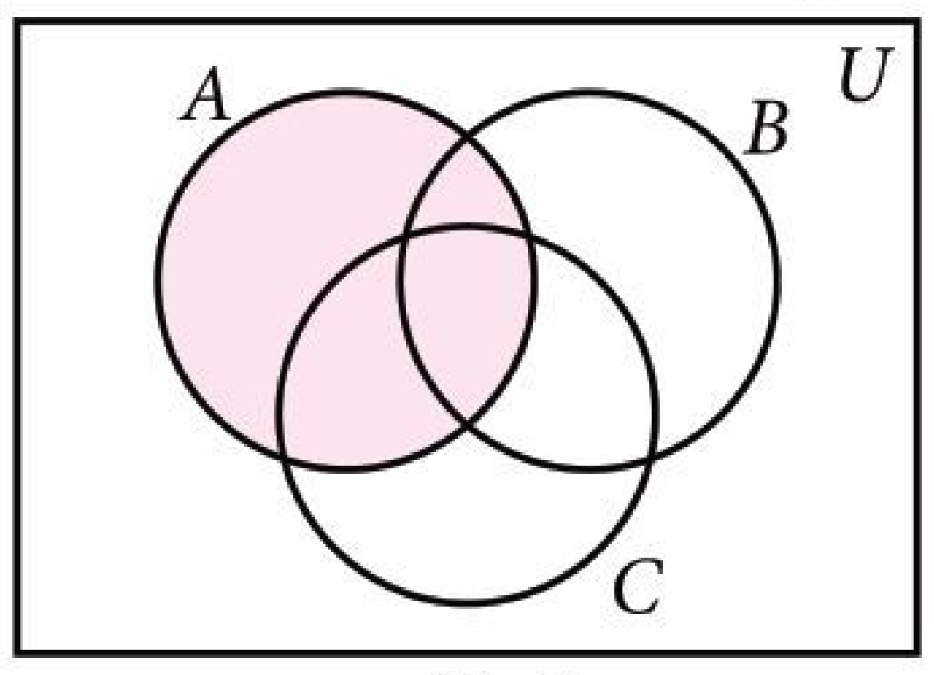
Now, $B \cap C = \{1, 4, 9\} \cap \{1, 8, 27\} = \{1\}$

$\Rightarrow \quad A \cap (B \cap C) = \{2, 3\} \cap \{1\} = \phi$

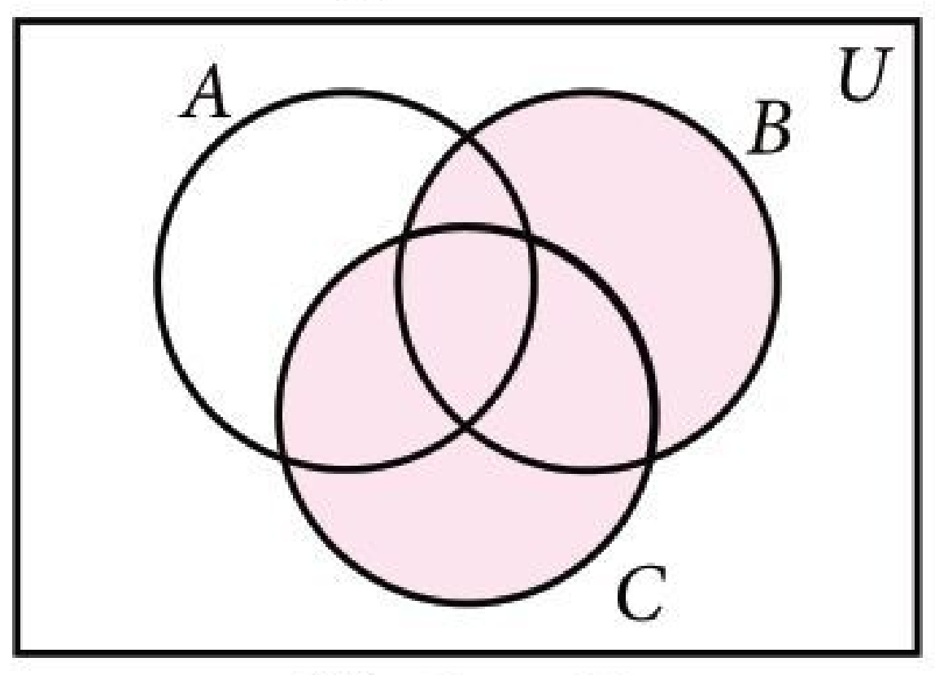
$\therefore \quad (A \cap B) \cap C = A \cap (B \cap C)$

OR

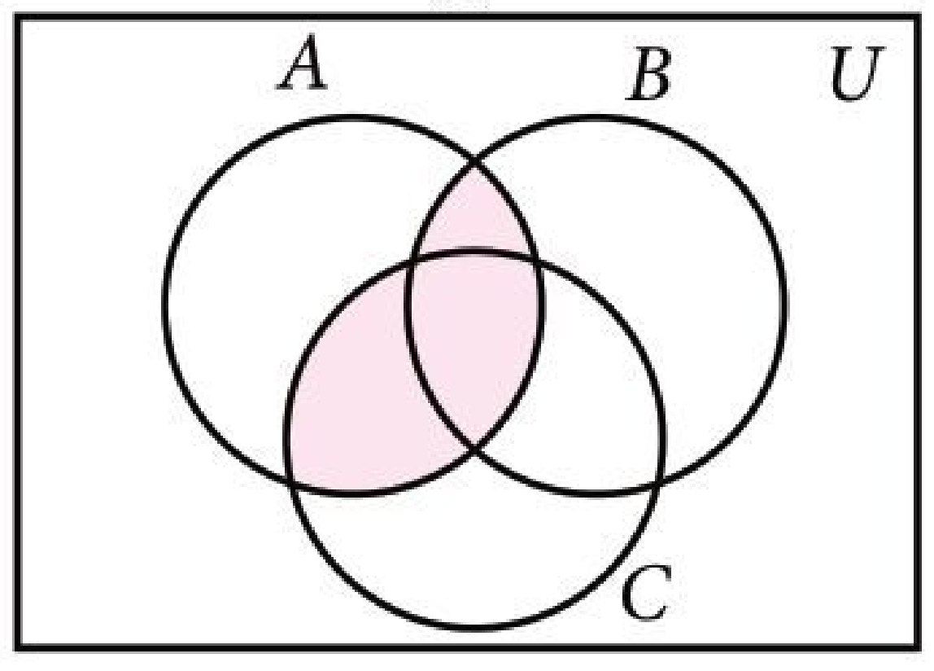
(a) The shaded area represents the given sets.



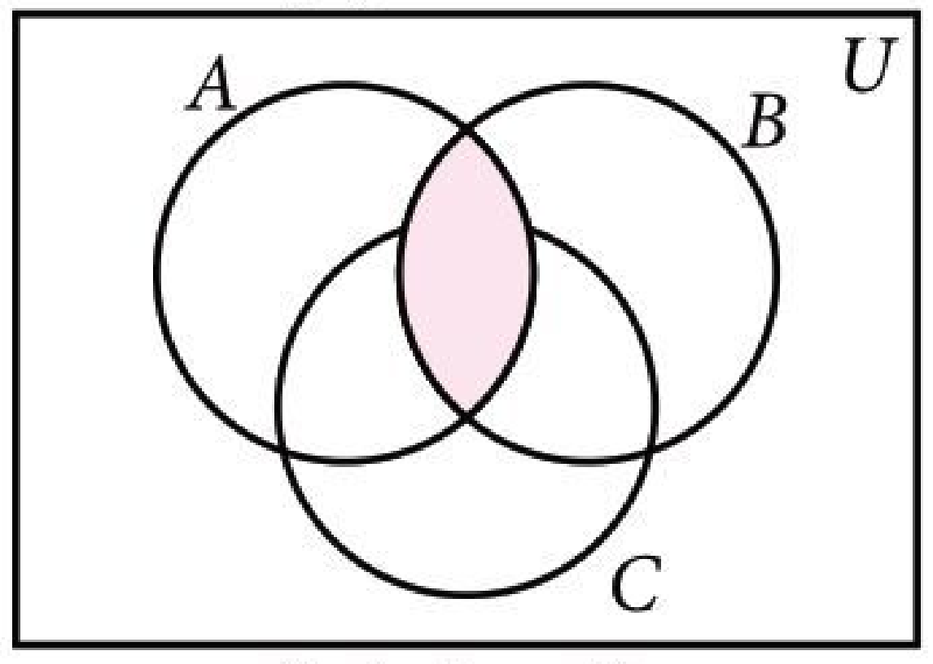
(i) A



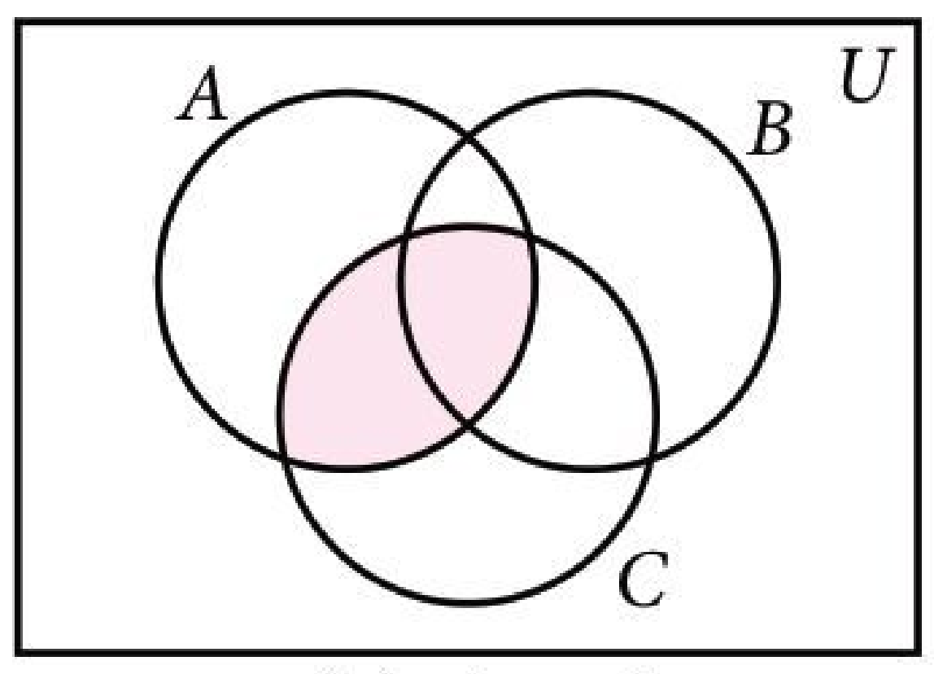
(ii) $B \cup C$



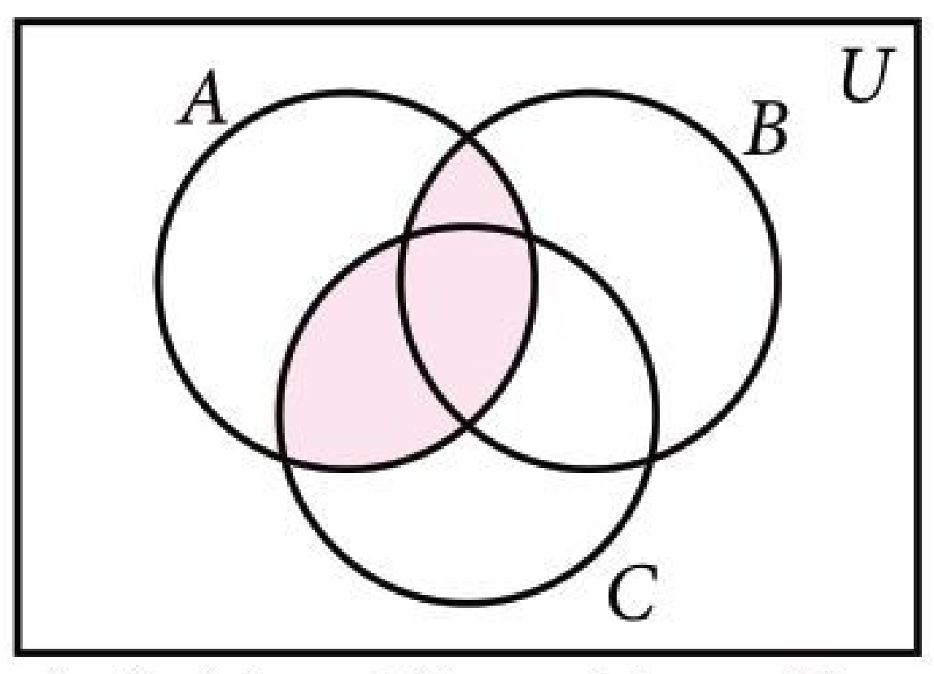
(iii) $A \cap (B \cup C)$



(iv) $A \cap B$



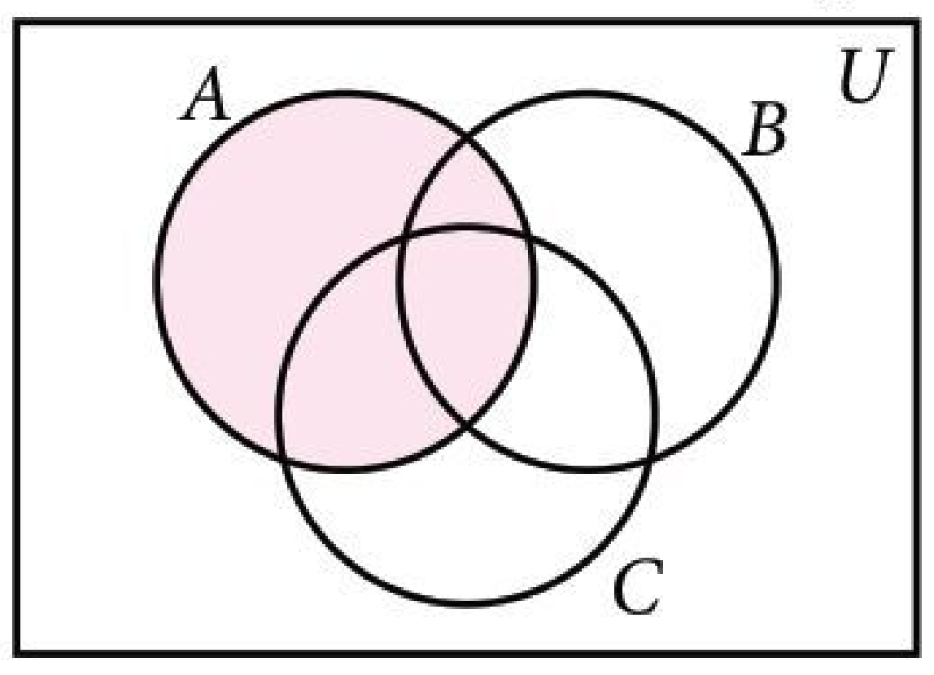
(v) $A \cap C$



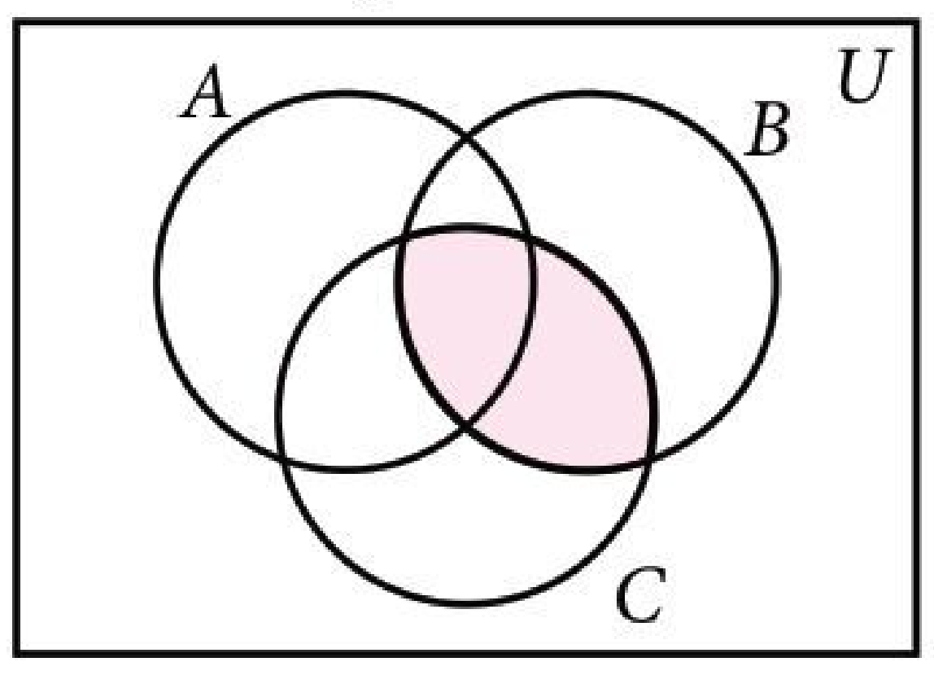
(vi) $(A \cap B) \cup (A \cap C)$

From (iii) and (vi), it is clear that
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

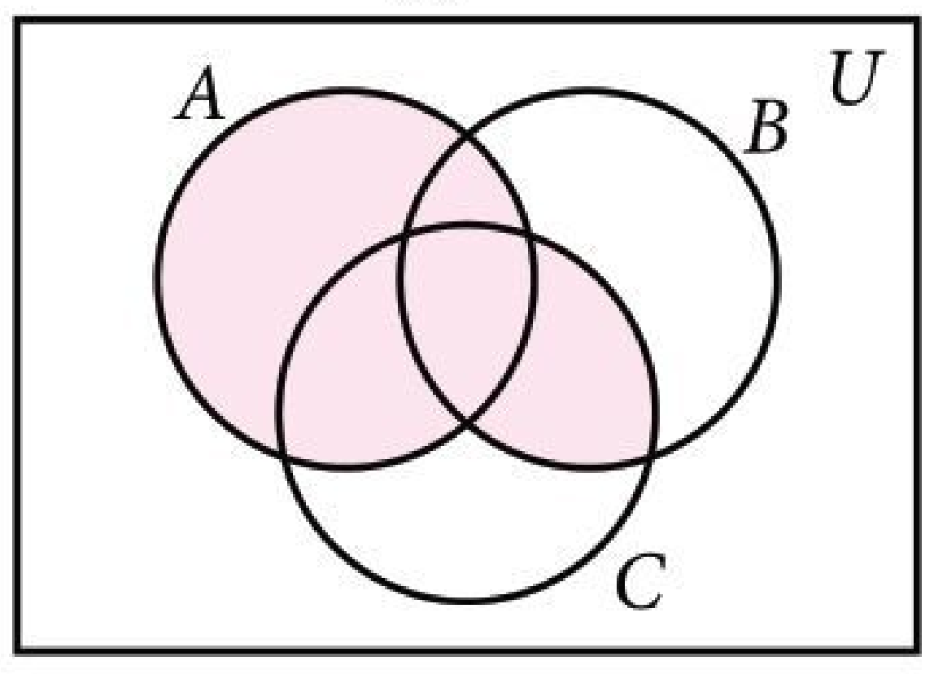
(b) The shaded area represents the given sets.



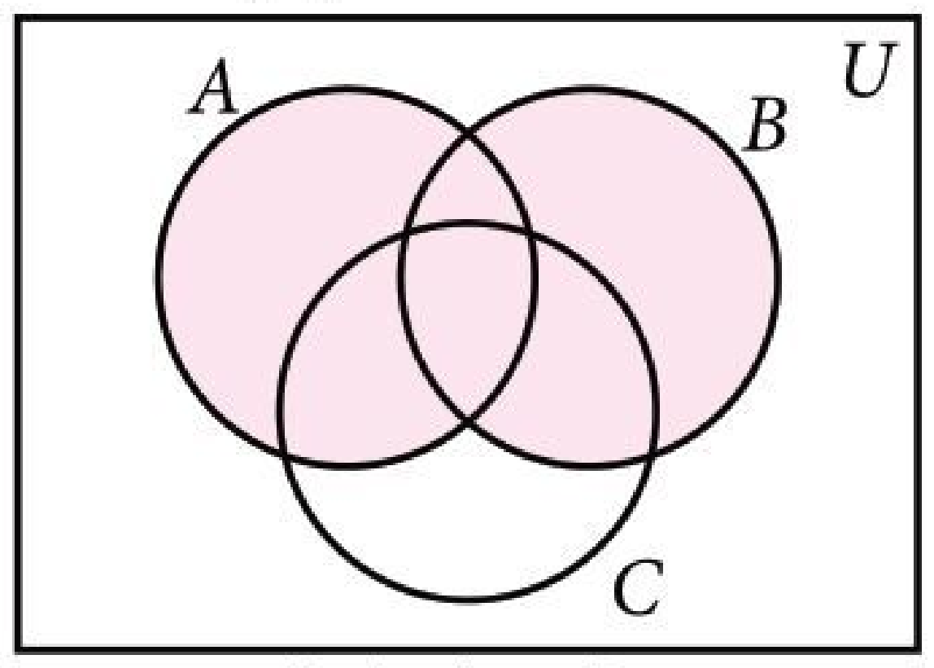
(i) A



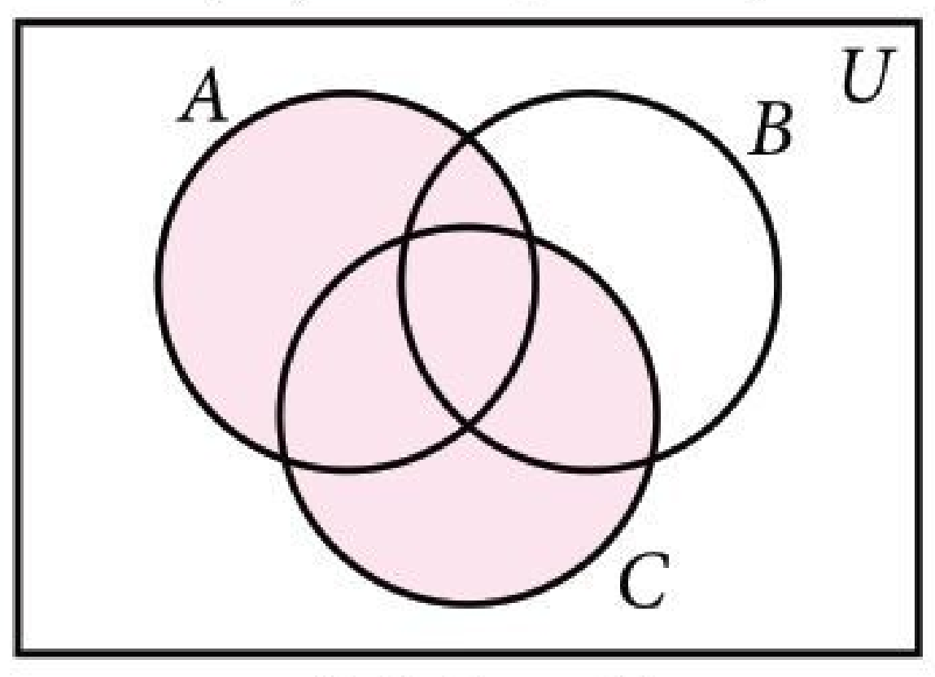
(ii) $B \cap C$



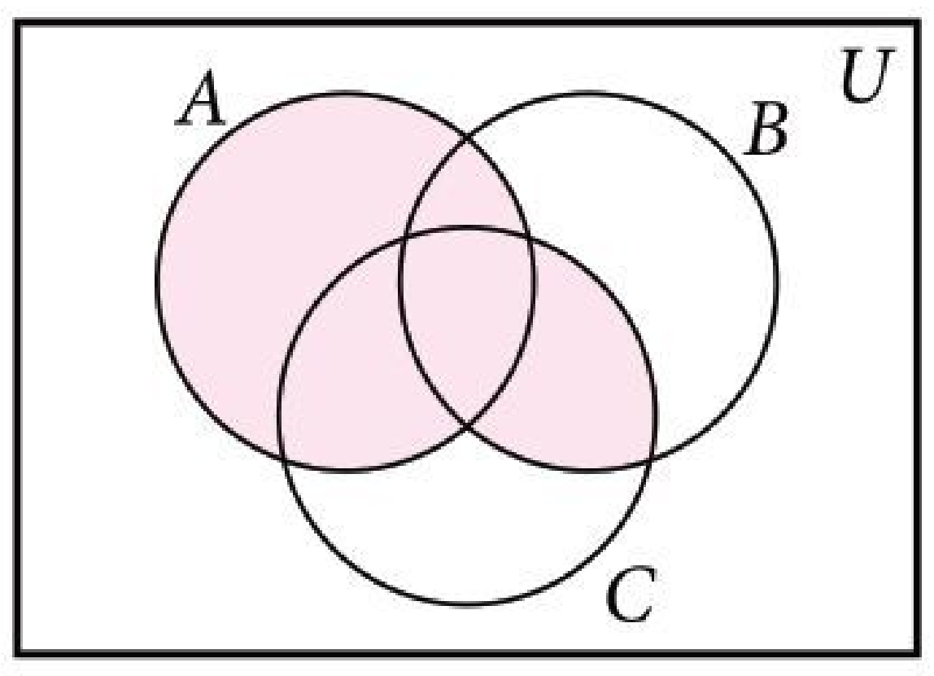
(iii) $A \cup (B \cap C)$



(iv) $A \cup B$



(v) $A \cup C$



(vi) $(A \cup B) \cap (A \cup C)$

From (iii) and (vi), it is clear that
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

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YOU ASK
WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. For what values of k will the equation $|z-z_1|^2+|z-z_2|^2=k, k\in R$, represent a circle?

(Ayush Garg, U.P.)

Ans. The given equation can be written as

$$2|z|^2+|z_1|^2+|z_2|^2-2\,\mathrm{Re}(z\bar{z}_1)-2\,\mathrm{Re}(z\bar{z}_2)=k$$

$$\Rightarrow\quad 2|z|^2-2\,\mathrm{Re}\{z(\bar{z}_1+\bar{z}_2)\}=k-(|z_1|^2+|z_2|^2)$$

$$\Rightarrow\quad |z|^2-\mathrm{Re}\{z(\bar{z}_1+\bar{z}_2)\}=\frac{1}{2}(k-|z_1|^2-|z_2|^2)$$

$$\Rightarrow\quad |z|^2-2\,\mathrm{Re}\left\{\frac{z(\bar{z}_1+\bar{z}_2)}{2}\right\}=\frac{1}{2}(k-|z_1|^2-|z_2|^2)$$

$$\Rightarrow\quad \left|z-\frac{z_1+z_2}{2}\right|^2-\frac{1}{4}|z_1+z_2|^2=\frac{1}{2}(k-|z_1|^2-|z_2|^2)$$

$$\Rightarrow\quad \left|z-\frac{z_1+z_2}{2}\right|^2=\frac{1}{2}k-\frac{1}{4}\{|z_1|^2+|z_2|^2-2\,\mathrm{Re}(z_1\bar{z}_2)\}$$

$$=\frac{1}{2}k-\frac{1}{4}|z_1-z_2|^2$$

$$\Rightarrow\quad \left|z-\frac{z_1+z_2}{2}\right|^2=\frac{1}{4}(2k-|z_1-z_2|^2),$$

which will represent a real circle having centre at $\frac{z_1+z_2}{2}$ and radius $\frac{1}{2}\sqrt{2k-|z_1-z_2|^2}$ provided $k\ge\frac{1}{2}|z_1-z_2|^2$.

2. The sum of the squares of three distinct real numbers in G.P., is S^2. If their sum is λS, show that

$\lambda^2\in\left(\frac{1}{3},1\right)\cup(1,3)$. *(Shashank Roy, West Bengal)*

Ans. Let three numbers in G.P. be a, ar, ar^2.
According to the given condition, we have

$$\lambda S=a+ar+ar^2 \quad\text{... (i)}$$

$$\text{and } S^2=a^2+a^2r^2+a^2r^4 \quad\text{... (ii)}$$

Squaring equation (i) and dividing by equation (ii), we have

$$\lambda^2=\frac{a^2(1+r+r^2)^2}{a^2(1+r^2+r^4)}=\left(\frac{1-r^3}{1-r}\right)^2\times\left(\frac{1-r^2}{1-r^6}\right)$$

$$=\frac{1-r^3}{1-r}\times\frac{1+r}{1+r^3}=\frac{1+r+r^2}{1-r+r^2}$$

$$\therefore\quad (\lambda^2-1)r^2-(\lambda^2+1)r+(\lambda^2-1)=0 \quad\text{...(iii)}$$

For real r, we have

$(\lambda^2+1)^2>4(\lambda^2-1)^2$ and $(\lambda^2-1)\ne 0$

Solving the first inequality, we have $x^2>4(x-2)^2$

[Putting $\lambda^2+1=x$]

$$\Rightarrow\quad 3x^2-16x+16<0$$

$$\Rightarrow\quad (x-4)\left(x-\frac{4}{3}\right)<0\quad\Rightarrow\frac{4}{3}<x<4$$

$$\Rightarrow\quad \frac{4}{3}<\lambda^2+1<4\Rightarrow\frac{1}{3}<\lambda^2<3\Rightarrow\lambda^2\in\left(\frac{1}{3},3\right)$$

From second inequality $\lambda^2\ne 1$

$$\therefore\quad \lambda^2\in\left(\frac{1}{3},1\right)\cup(1,3)$$

3. Let $[\sqrt{n^2+1}]=[\sqrt{n^2+\lambda}]$, where $n, \lambda\in N$ and $[\cdot]$ represents greatest integer function. Show that λ can have $2n$ different values. *(Tejaswin Jha, Bihar)*

Ans. We have, $n^2+1=(n+1)^2-2n<(n+1)^2\ \forall\ n\in N$

$$\Rightarrow\quad \sqrt{n^2+1}<n+1$$

Thus, we have, $n<\sqrt{n^2+1}<n+1$

$$\therefore\quad \left[\sqrt{n^2+1}\right]=n$$

Now, the given equation reduces to $\left[\sqrt{n^2+\lambda}\right]=n$

$$\Rightarrow\quad n<\sqrt{n^2+\lambda}<(n+1)\Rightarrow n^2<n^2+\lambda<(n+1)^2$$

$$\Rightarrow\quad 0<\lambda<2n+1.$$

Hence, λ can have $2n$ different values.

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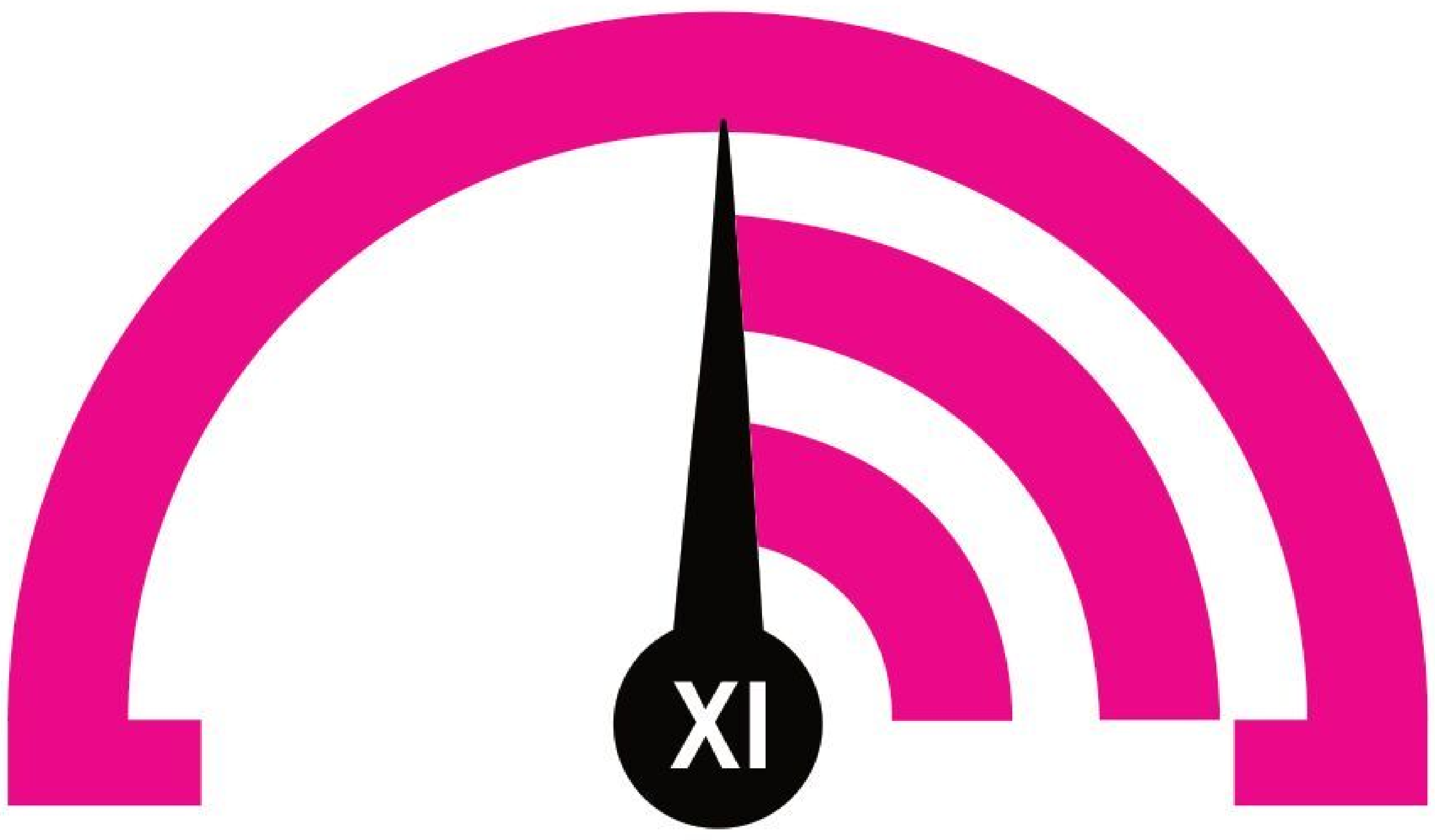
MONTHLY TEST
DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 1 : Sets, Relations & Functions

Time Taken : 60 Min.

Only One Option Correct Type

1. The domain of the function $f(x)=\sqrt{(2-|x|)}+\sqrt{(1+|x|)}$ is

(a) [2, 6] (b) (–2, 6)
(c) [8, 12] (d) [– 2, 2]

2. If A and B are two sets, then $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to

(a) $A \cup B$ (b) $A \cap B$
(c) A (d) B'

3. In a class, 70 students wrote two tests viz; test-I and test-II. 50% of the students failed in test-I and 40% of the students in test-II. How many students passed in both tests?

(a) 21 (b) 7 (c) 28 (d) 14

4. If $2f(x^2)+3f\left(\frac{1}{x^2}\right)=x^2-1\ \forall\ x\in R-\{0\}$,

then $5x^2f(x^2)$ is

(a) $(1 + 2x^2)(1 - 3x^2)$ (b) $(3 - 2x^2)(1 + x^2)$
(c) $(3 + 2x^2)(1 - x^2)$ (d) none of these

5. On its annual sports day, school awarded 35 medals in athletics, 15 in judo and 18 in swimming. If these medals goes to a total of 58 students and only three of them got medals in all the three sports. The number of students who received medals in exactly two of the three sports are

(a) 9 (b) 4 (c) 5 (d) 7

6. If $A = \{7, 8, 9\}$ and $B = \{9, 5\}$, then $(A\cup B) \times (A\cap B)$ is

(a) {(7, 9), (7, 5), (8, 9), (8, 5), (9, 9), (9, 5)}
(b) {(5, 9), (7, 9), (8, 9), (9, 9)}
(c) {(9, 5), (9, 7), (9, 8), (9, 9)}
(d) none of these

One or More Than One Option(s) Correct Type

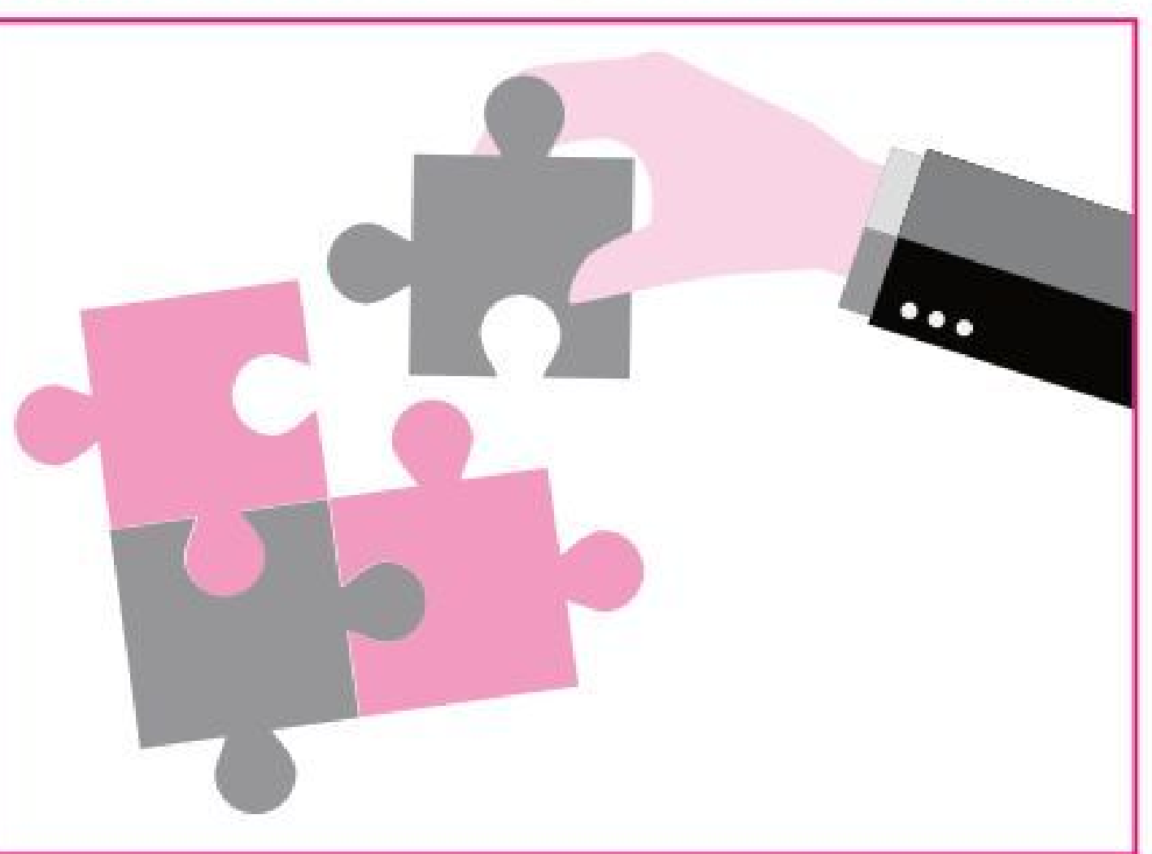
7. From 50 students taking examinations in Mathematics, Physics and Chemistry, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 passed Mathematics and Chemistry and at most 20 passed Physics and Chemistry. The largest possible number that could have passed all three examinations is

(a) 11 (b) 12 (c) 13 (d) 14

8. If $e^x + e^{f(x)} = e$, then for $f(x)$

(a) domain = $(-\infty, 1)$ (b) range = $(-\infty, 1)$
(c) domain = $(-\infty, 0]$ (d) range = $(-\infty, 1]$

PUZZLE CORNER



ANSWER - JULY 2022

2÷ 2	4	8+ 1	8+ 5	1– 3
1 1	5× 5	4	3	2
12+ 4	1	3	3– 2	5
5	6+ 3	2	1	4× 4
3	11+ 2	5	4	1

9. If $f(x)=\dfrac{x}{x^2+1}$ and $f(A)=\left\{y:-\dfrac{1}{2}\le y<0\right\}$, then set A is

(a) $[-1, 0]$ (b) $(-\infty, -1]$
(c) $(-\infty, 0)$ (d) $(-\infty, \infty)$

10. If $A = \{(x, y) : x^2 + y^2 \le 1, x, y \in R\}$ and $B = \{(x, y) : x^2 + y^2 \le 4;\ x, y \in R\}$, then

(a) $A - B = A$ (b) $B - A = B$
(c) $A - B = \phi$ (d) $B - A \ne \phi$

11. Let N be the set of natural numbers and for $a \in N$, aN denotes the set $\{ax : x \in N\}$. If $bN \cap cN = dN$, where b, c, d are natural numbers greater than 1 and the greatest common divisor (G.C.D.) of b and c is 1, then d equals

(a) $b + c$ (b) $\{b, c\}$
(c) $\min\{b, c\}$ (d) bc

12. Let Z denote the set of integers, then $\{x \in Z : |x - 3| < 4\} \cap \{x \in Z : |x - 4| < 5\} =$

(a) $\{-1, 0, 1, 2, 3, 4\}$ (b) $\{-1, 0, 1, 2, 3, 4, 5\}$
(c) $\{0, 1, 2, 3, 4, 5, 6\}$
(d) $\{-1, 0, 1, 2, 3, 5, 6, 7, 8, 9\}$

13. Let $A = \{a, b\}$, $B = \{c, d\}$ and $f : A \to B$ given by $f(x) = 3x^2 - 2$ and $g : A \to B$ given by $g(x) = 1 - 8x$, if $f = g$, then

(a) $A = \left\{-2, \dfrac{1}{2}\right\}$ (b) $B = \left\{24, \dfrac{-23}{3}\right\}$
(c) $A = \left\{-3, \dfrac{1}{3}\right\}$ (d) $B = \left\{25, \dfrac{-5}{3}\right\}$

Comprehension Type

In a class of 175 students the following data shows the number of students opting one or more subjects. Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18.

14. How many students have opted Mathematics alone?

(a) 60 (b) 35 (c) 70 (d) 48

15. How many students have opted Chemistry alone?

(a) 7 (b) 15 (c) 12 (d) 18

Matrix Match Type

16. Match the following :

	Column-I		Column-II
P.	The range of the function $f(x)=\dfrac{1}{2-\cos 3x}$ is	1.	a subset of $[0, 1]$
Q.	The range of the function $f(x)=\sqrt{(x-4)}+\sqrt{(6-x)}$ is	2.	a subset of $[1, 2]$
R.	If the function $f(x)$ is defined for $x \in [0, 1]$, then the function $f(2x+3)$ is defined for	3.	$\left[-\dfrac{3}{2}, -1\right]$
		4.	$\left[\dfrac{1}{3}, 1\right]$
		5.	$[\sqrt{2}, 2]$

	P	Q	R
(a)	1, 2, 4	3, 5	2
(b)	1, 4	2, 3	5
(c)	1, 4	2, 5	3
(d)	4, 2, 5	1, 3	1

Numerical Answer Type

17. A polynomial $f(x)$ satisfies the condition $f(x)\cdot f\left(\dfrac{1}{x}\right)=f(x)+f\left(\dfrac{1}{x}\right)$ and $f(10) = 1001$, then the value of $f(20)$ must be

18. If $n(A \cap B) = 10$, $n(B \cap C) = 20$ and $n(A \cap C) = 30$, then the greatest possible value of $n(A \cap B \cap C)$ is

19. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is

20. If $f(x) = a(x^n + 3)$ and $f(1) = 12$, $f(3) = 36$, then the value of $8f(2)$ must be

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Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

CLASS-XII

Series-1 Relations and Functions | Inverse Trigonometric Functions

Time allowed : 3 hours
Maximum marks : 80

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-*I *and* II.
2. *Section-I comprises of* 16 *very short answer type questions.*
3. *Section-II contains* 2 *case study-based questions.*

Part - B :

1. *It consists of three Sections-*III, IV *and* V.
2. *Section-*III *comprises of* 10 *questions of* 2 *marks each.*
3. *Section-*IV *comprises of* 7 *questions of* 3 *marks each.*
4. *Section-*V *comprises of* 3 *questions of* 5 *marks each.*
5. *Internal choice is provided in 3 questions of Section-*III, *2 questions of Section-*IV *and 3 questions of Section-*V. *You have to attempt only one of the alternatives in all such questions.*

PART - A

SECTION - I

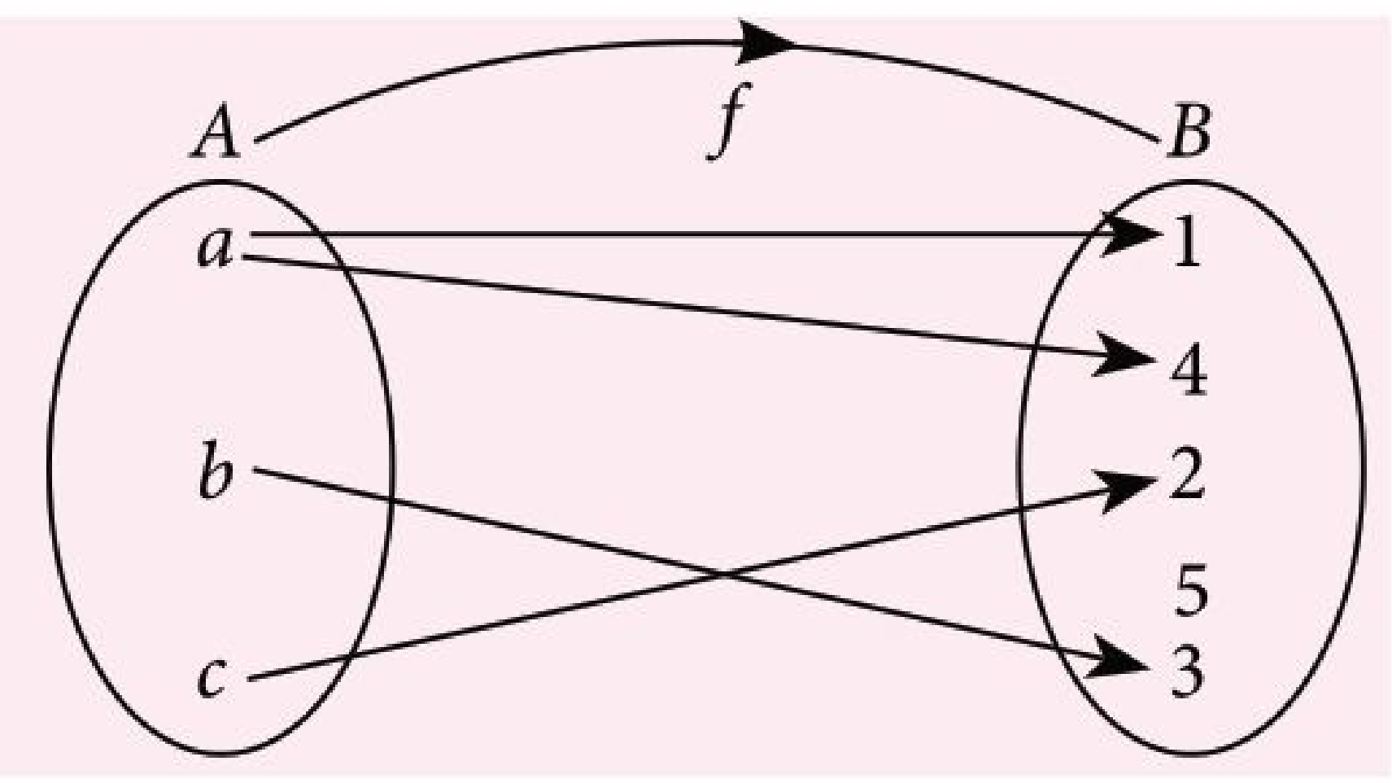
1. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R\beta \Leftrightarrow \alpha \perp \beta, \alpha, \beta \in L$. Then, show that R is symmetric relation.

2. Find the interval in which the function $f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$ is one-one.

3. The relation R is defined on the set of natural numbers as $\{(a, b): a = 2b\}$. Then find R^{-1}.

OR

Show that the given diagram is not a function.



4. Write the identity relation on set A = $\{11, 12, 13\}$.

5. Find the range of the function $f: [0, 1] \to R$ defined by $f(x) = x^3 - x^2 + 4x + 2\sin^{-1}x$.

6. Prove that the relation R defined on the set N of natural numbers by $xRy \Leftrightarrow 2x^2 - 3xy + y^2 = 0$ *i.e.*, $R = \{(x, y) : x, y \in N \text{ and } 2x^2 - 3xy + y^2 = 0\}$ is not symmetric.

7. Find the value of $\sin^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\frac{1}{2}$.

OR

Find the domain of the function defined by $f(x) = \sin^{-1}x + \cos x$.

8. Evaluate : $\cos\left\{\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\frac{\pi}{6}\right\}$

9. Evaluate : $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

10. Determine if the function given below is one-to-one or not.

"To each person on earth assign the number which corresponds to his height".

OR

A relation R in $S = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which element(s) of relation R be removed to make R an equivalence relation?

11. Find domain of $\cos^{-1}[x]$.

12. Evaluate : $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right]$.

13. Write whether $f : R \rightarrow R$, given by $f(x)=x+\sqrt{x^2}$ is one-one or many one function.

OR

State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive.

14. Let R be a relation in the set of natural numbers N defined by $R = \{(a, b) \in N \times N : a < b\}$. Is relation R reflexive? Give reason.

15. Find the domain of $\sec^{-1}(2x + 1)$.

16. If $A = \{0, 1\}$ and N be the set of natural numbers then show that the mapping $f : N \rightarrow A$ defined by $f(2n - 1) = 0, f(2n) = 1, \forall\ n \in N$, is onto.

OR

Let $A = \{1, 2, 3, ..., n\}$ and $B = \{a, b\}$. Then find the number of surjections from A to B.

SECTION - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub part carries 1 mark.

17. Consider the mapping $f : A \rightarrow B$ is defined by $f(x) = \frac{x-1}{x-2}$ such that f is a bijection.

Based on the above information, answer the following questions.

(i) Domain of f is

(a) $R - \{2\}$ (b) R

(c) $R - \{1, 2\}$ (d) $R - \{0\}$

(ii) Range of f is

(a) R (b) $R - \{1\}$

(c) $R - \{0\}$ (d) $R - \{1, 2\}$

(iii) If $g : R - \{2\} \rightarrow R - \{1\}$ is defined by $g(x) = 2f(x) - 1$, then $g(x)$ in terms of x is

(a) $\frac{x+2}{x}$ (b) $\frac{x+1}{x-2}$

(c) $\frac{x-2}{x}$ (d) $\frac{x}{x-2}$

(iv) The function g defined above, is

(a) One-one (b) Many-one

(c) Into (d) None of these

(v) A function $f(x)$ is said to be one-one iff

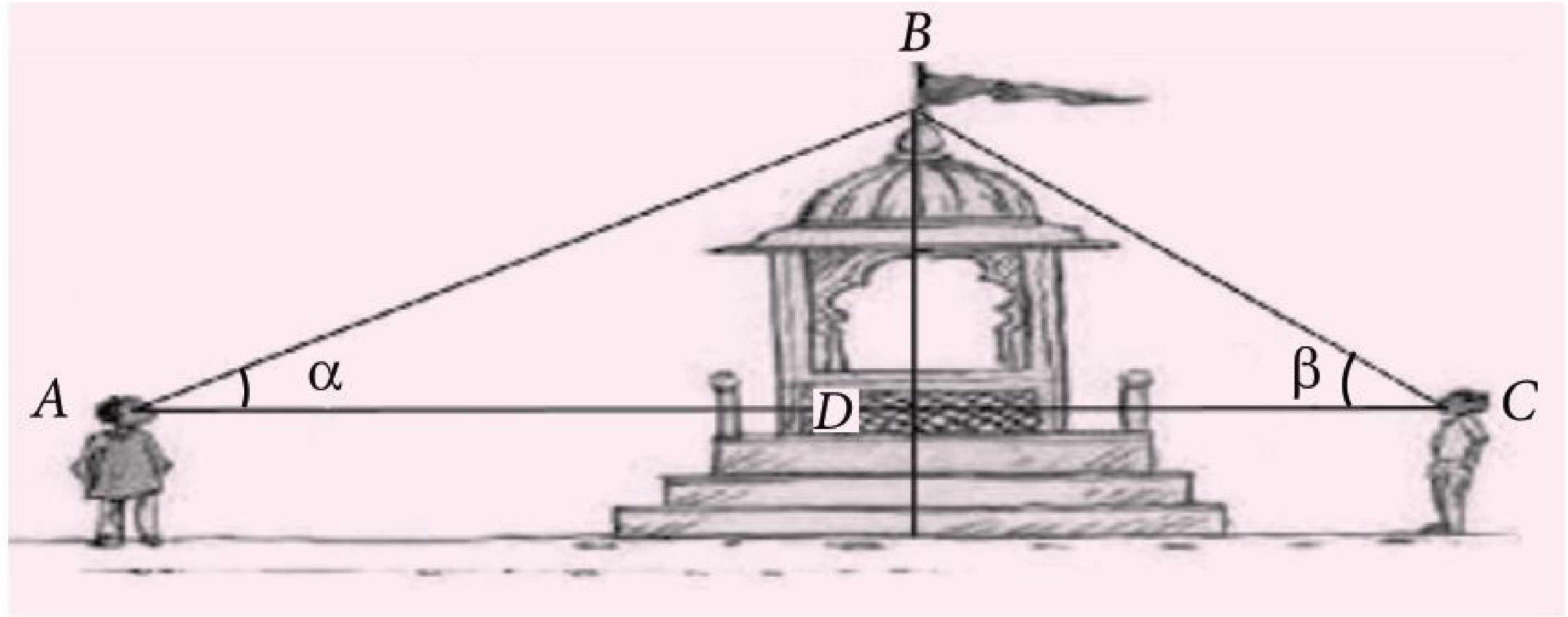
(a) $f(x_1) = f(x_2) \Rightarrow -x_1 = x_2$

(b) $f(-x_1) = f(-x_2) \Rightarrow -x_1 = x_2$

(c) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

(d) None of these

18. Two men on either side of a temple, which is 30 metres high above the stairs, observe its top at the angles of elevation α and β respectively (as shown in the figure below). The distance between the two men is $40\sqrt{3}$ metres and the distance between the first person A and the temple is $30\sqrt{3}$ metres.



Based on the above information, answer the following questions.

(i) $\angle CAB = \alpha =$

(a) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (b) $\sin^{-1}\left(\frac{1}{2}\right)$

(c) $\sin^{-1}(2)$ (d) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(ii) $\angle CAB = \alpha =$

(a) $\cos^{-1}\left(\frac{1}{5}\right)$ (b) $\cos^{-1}\left(\frac{2}{5}\right)$

(c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\cos^{-1}\left(\frac{4}{5}\right)$

(iii) $\angle BCA = \beta =$

(a) $\tan^{-1}\left(\frac{1}{2}\right)$ (b) $\tan^{-1}(2)$

(c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\tan^{-1}\left(\sqrt{3}\right)$

(iv) $\angle ABC =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(v) Domain and range of $\cos^{-1} x$ are respectively

(a) $(-1, 1), (0, \pi)$ (b) $[-1, 1], (0, \pi)$

(c) $[-1, 1], [0, \pi]$ (d) $(-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

PART - B

SECTION - III

19. Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class of 0.

20. Find the minimum value of n for which $\tan^{-1}\frac{n}{\pi} > \frac{\pi}{4}, n \in N$.

OR

Prove that : $\frac{1}{2}\tan^{-1} x = \cos^{-1}\left(\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}\right)$

21. If $(\sin^{-1}x)^2 + (\sin^{-1}y)^2 + (\sin^{-1}z)^2 = \frac{3}{4}\pi^2$, then find the value of $x^2 + y^2 + z^2$.

22. Write the function $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), x > 1$ in the simplest form.

23. Show that the fractional part function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x - [x]$ is neither one-one nor onto.

OR

Show that the signum function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

is neither one-one nor onto.

24. Find the value of $\tan\frac{1}{2}\left(\cos^{-1}\frac{\sqrt{5}}{3}\right)$.

OR

Simplify : $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right), \frac{\pi}{4} < x < \frac{5\pi}{4}$

25. Let $A = \{a, b, c\}$. How many relations can be defined in the set and how many of these are reflexive?

OR

Let $f : R \to R$ be defined by (i) $f(x) = x + |x|$ (ii) $f(x) = x + 1$. Determine whether or not f is onto.

26. If $\cot^{-1}\left(-\frac{1}{5}\right) = x$, then find the values of $\sin x$ and $\cos x$.

27. Determine whether the relation R in the set A of human beings in a city at a particular time defined by $R = \{(x, y) : x \text{ is wife of } y\, ; x, y \in A\}$ is reflexive and symmetric.

28. Prove that : $\cos^{-1} x + \cos^{-1}\left(\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right) = \frac{\pi}{3}$

SECTION - IV

29. Find the value of

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{\pi}{2}\right)\right].$$

30. Let $A = \left\{x : x \in R, -\frac{\pi}{2} \le x \le \frac{\pi}{2}\right\}$ and $B = \{y : y \in R, -1 \le y \le 1\}$. Show that the function $f : A \to B$ such that $f(x) = \sin x$ is bijective.

31. Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

OR

Show that the relation R defined by $(a, b)\, R\, (c, d)$ iff $a + d = b + c$ on the set $N \times N$ is an equivalence relation.

32. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto ? Justify your answer.

33. Using principal values, find the value of

$$\cot^{-1}\left(\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)\right).$$

34. If $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, $\forall\, xy > 1$ and

$$\tan^{-1}\left(\frac{1}{1+1\cdot 2}\right) + \tan^{-1}\left(\frac{1}{1+2\cdot 3}\right) + \ldots$$

$$+\tan^{-1}\left(\frac{1}{1+n\cdot(n+1)}\right) = \tan^{-1}\theta,$$ then find the value of θ.

OR

Prove that :

$$\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}$$

35. Show that the relation R on the set $\mathbb{R}$ of all real numbers, defined as $R = \{(a, b) : a \le b^2\}$ is neither reflexive nor symmetric nor transitive.

SECTION - V

36. Show that the relation R, defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides$\}$, is an equivalence relation on A. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

OR

Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\, R(c, d)$ if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

37. Show that the function $f : N \to N$ given by, $f(n) = n - (-1)^n$ for all $n \in N$ is a bijection.

OR

Show that the function $f : (-\infty, 0) \to (-1, 0)$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto.

38. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $a, x \in\ ]0, 1[$, then find the value of x.

OR

Prove that :

$$\cos^{-1}\left(\frac{\cos\alpha + \cos\beta}{1+\cos\alpha\cos\beta}\right) = 2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right).$$

SOLUTIONS

1. Given, $\alpha R \beta \Leftrightarrow \alpha \perp \beta$

$\therefore\ \alpha \perp \beta \Leftrightarrow \beta \perp \alpha \Rightarrow \beta R \alpha$

Hence, R is symmetric.

2. $f(x) = 2\sin\left(2x - \frac{\pi}{6}\right) + 4$

Since, $\sin x$ is one-one in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore\ -\frac{\pi}{2} \le 2x - \frac{\pi}{6} \le \frac{\pi}{2} \Rightarrow x \in \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$

$\therefore f(x)$ is one-one in $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$

3. $R = \{(2, 1), (4, 2), (6, 3), \ldots\}$

So, $R^{-1} = \{(1, 2), (2, 4), (3, 6), \ldots.\}$

OR

As $f(a)$ is not unique, thus f is not a function.

4. Given, $A = \{11, 12, 13\}$.

$\therefore\ I_A = \{(11, 11), (12, 12), (13, 13)\}$

5. We have, $f(x) = x^3 - x^2 + 4x + 2\sin^{-1}x$

For $x = 0$, $f(0) = 0 + 2\sin^{-1}(0) = 0$

For $x = 1$, $f(1) = 1 - 1 + 4 + 2\sin^{-1}(1) = 4 + 2 \times \frac{\pi}{2} = 4 + \pi$

Since, $f(x)$ is an increasing function.

$\therefore$ Its range is $[0, 4 + \pi]$.

6. For $x = 1, y = 2$; $2x^2 - 3xy + y^2 = 2.1^2 - 3\cdot 1\cdot 2 + 2^2 = 0$

$\therefore$ $1R2$ *i.e.*, $(1, 2) \in R$

For $x = 2, y = 1$, $2\cdot 2^2 - 3\cdot 2\cdot 1 + 1^2 = 3 \ne 0$

So, $(2, 1) \notin R$

$\therefore$ R is not symmetric.

7. $\sin^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{1}{2} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}.$

OR

Domain of $\cos x$ is R and domain of $\sin^{-1} x$ is $[-1, 1]$. Therefore, the domain of $\cos x + \sin^{-1}x$ is $R \cap [-1, 1]$ *i.e.*, $[-1, 1]$.

8. $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right] = -1$

9. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) \ne \frac{3\pi}{4}$, because $\frac{3\pi}{4}$ does not lie between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

$$\text{Now, } \tan^{-1}\left(\tan\frac{3\pi}{4}\right)=\tan^{-1}\left\{\tan\left(\pi-\frac{\pi}{4}\right)\right\}$$

$$=\tan^{-1}\left(-\tan\frac{\pi}{4}\right)=-\frac{\pi}{4}$$

10. Since more than one person may have same height, therefore the function defined by $f(x) = y$, where x is a person and y is his height, is not a one-one function.

OR

We have, $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$ which is reflexive and transitive.

For R to be symmetric $(1, 2)$ should be removed from R.

11. Since $0 \le \cos^{-1}[x] \le \pi$

$\Rightarrow\ -1 \le [x] \le 1 \Rightarrow -1 \le x < 2 \Rightarrow x \in [-1, 2)$

12. We have, $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$

$$=\sin\left[\frac{\pi}{3}-\sin^{-1}\left(\sin\left(\frac{-\pi}{6}\right)\right)\right]$$

$$=\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)=\sin\frac{\pi}{2}=1$$

13. Here, $f: R \to R$ given by $f(x)=x+\sqrt{x^2}$

$= x \pm x = 0$ or $2x$

Now, $f(0) = 0$ and $f(-1)=-1+\sqrt{(-1)^2}=-1+1=0$

So, image of 0 and –1 is 0.

$\therefore$ f is many-one function.

OR

We have, $R = \{(1, 2), (2, 1)\}$ defined on set $\{1, 2, 3\}$.

As, $(1, 2) \in R$, $(2, 1) \in R$ but $(1, 1) \notin R$.

$\therefore$ R is not transitive.

14. Given, $R = \{(a, b) \in N \times N : a < b\}$

No, R is not reflexive as for $(a, a) \in R\ a \not< a$.

15. $\sec^{-1}(2x + 1)$ is defined if $2x + 1 \ge 1$ or $2x + 1 \le -1$

$\Rightarrow\ 2x \ge 0$ or $2x \le -2 \Rightarrow\ x \ge 0$ or $x \le -1$

$\Rightarrow\ x \in (-\infty, -1] \cup [0, \infty)$

Hence, the domain of $\sec^{-1}(2x + 1)$ is $(-\infty, -1] \cup [0, \infty)$

16. Given, $A = \{0, 1\}$

Since, $f(2n - 1) = 0, f(2n) = 1\ \forall\ n \in N$

$\therefore$ A = range

Hence, the mapping $f : N \to A$ is onto.

OR

Given that, $A = \{1, 2, 3, ..., n\}$ and $B = \{a, b\}$.

Since, every element of domain A has two choices *i.e.*, a or b.

$\therefore$ Number of functions will be 2^n.

Also, there will be two cases:

(i) When all the elements of A will map to a only, then b is left which do not have any pre-image in A.

(ii) When all the elements of A will map to b only, then a is left which do not have any pre-image in A.

Thus in both cases, function is not onto.

$\therefore$ Total number of onto functions $= 2^n - 2$.

17. **(i) (a) :** For $f(x)$ to be defined $x - 2 \ne 0$ *i.e.*, $x \ne 2$

$\therefore$ Domain of $f = R - \{2\}$

(ii) (b) : Let $y = f(x)$, then $y=\frac{x-1}{x-2}$

$\Rightarrow\ xy - 2y = x - 1\ \Rightarrow\ xy - x = 2y - 1\ \Rightarrow\ x=\frac{2y-1}{y-1}$

Since, $x \in R - \{2\}$, therefore $y \ne 1$

Hence, range of $f = R - \{1\}$

(iii) (d) : We have, $g(x) = 2f(x) - 1$

$$=2\left(\frac{x-1}{x-2}\right)-1=\frac{2x-2-x+2}{x-2}=\frac{x}{x-2}$$

(iv) (a) : We have, $g(x)=\frac{x}{x-2}$

Let $g(x_1) = g(x_2)\ \Rightarrow\ \frac{x_1}{x_1-2}=\frac{x_2}{x_2-2}$

$\Rightarrow\ x_1x_2 - 2x_1 = x_1x_2 - 2x_2$

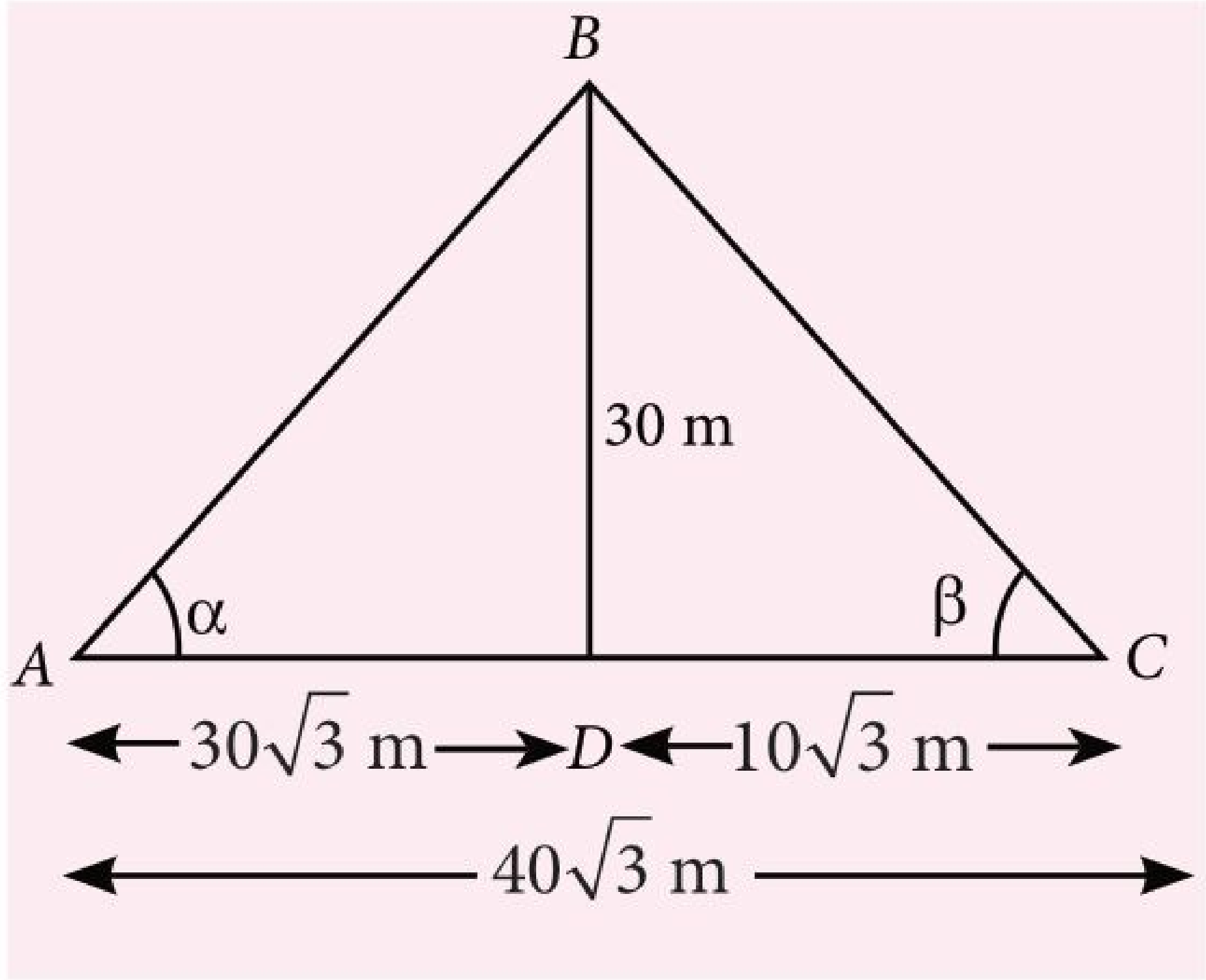
$\Rightarrow\ 2x_1 = 2x_2\ \Rightarrow\ x_1 = x_2$

Thus, $g(x_1) = g(x_2)\ \Rightarrow\ x_1 = x_2$

Hence, $g(x)$ is one-one.

(v) (c)

18. **(i) (b) :** We have, $BD = 30$ m, $AC=40\sqrt{3}$ m and $AD=30\sqrt{3}$ m



Clearly, $BD \perp AC$

∴ In right ΔADB, $AB^2 = AD^2 + BD^2$

$$= (30\sqrt{3})^2 + (30)^2 = 3600$$

$\Rightarrow AB = 60$ m

Now, $\sin\alpha = \dfrac{DB}{AB} = \dfrac{30}{60} = \dfrac{1}{2}$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \angle CAB = \alpha = \sin^{-1}\left(\frac{1}{2}\right)$$

(ii) (c) : In right ΔADB, $\cos\alpha = \dfrac{AD}{AB} = \dfrac{30\sqrt{3}}{60} = \dfrac{\sqrt{3}}{2}$

$$\Rightarrow \angle CAB = \alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

(iii) (d) : Clearly, $CD = AC - AD = 40\sqrt{3} - 30\sqrt{3} = 10\sqrt{3}$ m

$$\therefore \quad \tan\beta = \frac{BD}{CD} = \frac{30}{10\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \angle BCA = \beta = \tan^{-1}(\sqrt{3})$$

(iv) (c) : $\sin\alpha = \dfrac{1}{2} \Rightarrow \alpha = 30°$

$\tan\beta = \sqrt{3} \Rightarrow \beta = 60°$

$$\therefore \angle ABC = 180° - (\alpha + \beta) = 180° - (30° + 60°) = 90°$$

or $\angle ABC = \dfrac{\pi}{2}$

(v) (c) : Domain and range of $\cos^{-1}x$ are $[-1, 1]$ and $[0, \pi]$ respectively.

19. Here, $R = \{(a, b) \in A \times A : 2 \text{ divides } (a - b)\}$

This is the given equivalence relation, where

$A = \{0, 1, 2, 3, 4, 5\}$ ∴ $[0] = \{0, 2, 4\}$.

20. We have, $\tan^{-1}\dfrac{n}{\pi} > \dfrac{\pi}{4} \Rightarrow \tan^{-1}\dfrac{n}{\pi} > \tan^{-1} 1$

$$\Rightarrow \tan\left(\tan^{-1}\frac{n}{\pi}\right) > \tan(\tan^{-1} 1)$$

[∵ $\tan\theta$ is an increasing function]

$$\Rightarrow \frac{n}{\pi} > 1 \Rightarrow n > \pi \cong 3.14 \Rightarrow n = 4, 5, 6, \ldots$$

Hence, the minimum value of n is 4.

OR

Let $\tan^{-1} x = y \Rightarrow x = \tan y$

$$\text{R.H.S.} = \cos^{-1}\left(\sqrt{\frac{1+\sqrt{1+\tan^2 y}}{2\sqrt{1+\tan^2 y}}}\right) = \cos^{-1}\left(\sqrt{\frac{1+\sec y}{2\sec y}}\right)$$

$$= \cos^{-1}\left(\sqrt{\frac{1+\cos y}{2}}\right) = \cos^{-1}\left(\sqrt{\frac{2\cos^2\frac{y}{2}}{2}}\right)$$

$$= \cos^{-1}\left(\cos\frac{y}{2}\right) = \frac{y}{2} = \frac{1}{2}\tan^{-1} x = \text{L.H.S.}$$

21. Since, we know that $\sin^{-1}x \le \dfrac{\pi}{2}, x \in [-1, 1]$

$$\Rightarrow (\sin^{-1}x)^2 \le \frac{\pi^2}{4}, x \in [-1, 1] \quad \ldots\text{(i)}$$

$$\text{Similarly, } (\sin^{-1}y)^2 \le \frac{\pi^2}{4}, y \in [-1, 1] \quad \ldots\text{(ii)}$$

$$\text{and } (\sin^{-1}z)^2 \le \frac{\pi^2}{4}, z \in [-1, 1] \quad \ldots\text{(iii)}$$

$$\text{Now, given } \frac{3\pi^2}{4} = (\sin^{-1}x)^2 + (\sin^{-1}y)^2 + (\sin^{-1}z)^2$$

$$\le \frac{3\pi^2}{4} \qquad \text{[From (i), (ii) and (iii)]}$$

$$\Rightarrow \sin^{-1}x = \pm\frac{\pi}{2}, \sin^{-1}y = \pm\frac{\pi}{2}, \sin^{-1}z = \pm\frac{\pi}{2}$$

$$\Rightarrow x = \pm 1, y = \pm 1, z = \pm 1$$

$$\therefore x^2 + y^2 + z^2 = 1^2 + 1^2 + 1^2 = 3$$

22. Let $x = \sec y \Rightarrow y = \sec^{-1} x, x > 1, 0 < y < \dfrac{\pi}{2}$

$$\text{Now, } \sqrt{x^2 - 1} = \sqrt{\sec^2 y - 1} = \sqrt{\tan^2 y} = |\tan y| = \tan y$$

$$\left(\because 0 < y < \frac{\pi}{2} \Rightarrow \tan y > 0 \Rightarrow |\tan y| = \tan y\right)$$

$$\therefore \cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \cot^{-1}\left(\frac{1}{\tan y}\right) = \cot^{-1}(\cot y)$$

$= y = \sec^{-1}x.$

23. Since $f\left(\dfrac{1}{2}\right) = \dfrac{1}{2} - \left[\dfrac{1}{2}\right] = \dfrac{1}{2} - 0 = \dfrac{1}{2}$

and $f\left(\dfrac{3}{2}\right) = \dfrac{3}{2} - \left[\dfrac{3}{2}\right] = \dfrac{3}{2} - 1 = \dfrac{1}{2}$, so the two different elements $\dfrac{1}{2}, \dfrac{3}{2} \in R$ (domain of f) have same image.

⇒ f is not one-one.

Also, range of $f = [0, 1)$, which is a proper subset of $\mathbb{R}$ (codomain of f)

⇒ f is not onto.

OR

As $f(x) = 1$ for all $x > 0 \Rightarrow f(1) = 1$ and $f(2) = 1$.

So, the two different elements $1, 2 \in R$ (domain of f) have same image.

⇒ f is not one-one.

Also, range of $f = \{-1, 0, 1\}$, which is a proper subset of $\mathbb{R}$ (codomain of f)

$\Rightarrow$ f is not onto.

24. Let $\cos^{-1}\frac{\sqrt{5}}{3} = \theta$. Then, $\cos\theta = \frac{\sqrt{5}}{3}$

$$\therefore \quad \tan\frac{1}{2}\left(\cos^{-1}\frac{\sqrt{5}}{3}\right) = \tan\frac{\theta}{2} = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}$$

$$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \left\{\frac{\left(1-\frac{\sqrt{5}}{3}\right)}{\left(1+\frac{\sqrt{5}}{3}\right)}\right\}^{1/2} = \left(\frac{3-\sqrt{5}}{3+\sqrt{5}}\right)^{1/2}$$

$$= \left\{\frac{3-\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}\right\}^{1/2} = \frac{(3-\sqrt{5})}{2}.$$

OR

We have, $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)$$

$$= \sin^{-1}\left\{\sin\left(x+\frac{\pi}{4}\right)\right\} = \pi - \left(x+\frac{\pi}{4}\right)$$

$$\left[\because \frac{\pi}{4} < x < \frac{5\pi}{4} \Rightarrow \frac{\pi}{2} < x + \frac{\pi}{4} < \frac{3\pi}{2} \Rightarrow -\frac{\pi}{2} < \pi - \left(x+\frac{\pi}{4}\right) < \frac{\pi}{2}\right]$$

$$= \frac{3\pi}{4} - x$$

25. Total number of relations that can be defined on set A consisting of 3 elements to itself $= 2^{n^2}$

$= 2^{3^2} = 2^9 = 512.$

Number of reflexive relations on set A having 3 elements $= 2^{3(3-1)} = 2^6 = 64.$

OR

(i) : $f(x) = x + |x|$

$$= \begin{cases} x + x, \text{ if } x \ge 0 \\ x - x, \text{ if } x < 0 \end{cases} = \begin{cases} 2x, \text{ if } x \ge 0 \\ 0, \text{ if } x < 0 \end{cases}$$

Thus, $f(x) = 2x \ge 0 \ \forall\ x \ge 0$ and $f(x) = 0 \ \forall\ x < 0$

$\therefore$ $f(x)$ can't be negative for any $x \in R$

Thus, f is not onto.

(ii) We have $f(x) = x + 1 \ \forall\ x \in R$

For any $y \in R$, $y = f(x) \Rightarrow y = x + 1 \Rightarrow x = y - 1$

$\therefore$ $f(y - 1) = y - 1 + 1 = y$

Hence, f is onto.

26. Given, $\cot^{-1}\left(-\frac{1}{5}\right) = x \Rightarrow \cot x = -\frac{1}{5}, 0 < x < \pi$

$$\therefore \ \operatorname{cosec} x = \sqrt{1+\cot^2 x} = \sqrt{1+\left(-\frac{1}{5}\right)^2} = \frac{\sqrt{26}}{5}$$

[$\because$ cosec x is positive]

$$\Rightarrow \ \sin x = \frac{5}{\sqrt{26}}$$

Also, $\cos x = \cot x \cdot \sin x = \left(-\frac{1}{5}\right)\frac{5}{\sqrt{26}} = -\frac{1}{\sqrt{26}}$

27. The relation R on the set A of human beings is defined by $R = \{(x, y) : x \text{ is wife of } y\}$.

Consider any $x \in A$, then x is not the wife of x.

$\Rightarrow$ $(x, x) \notin R \Rightarrow R$ is not reflexive.

If $x, y \in A$ are such that $(x, y) \in R$ *i.e.*, x is wife of y, then y cannot be wife of x.

$\Rightarrow$ $(y, x) \notin R \Rightarrow R$ is not symmetric.

28. Let $\cos^{-1} x = \theta$

Then, $x = \cos\theta$ and $\sqrt{1-x^2} = \sin\theta$

$$\therefore \ \text{L.H.S.} = \cos^{-1}x + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)$$

$$= \theta + \cos^{-1}\left(\cos\frac{\pi}{3}\cos\theta + \sin\frac{\pi}{3}\sin\theta\right)$$

$$= \theta + \cos^{-1}\left\{\cos\left(\frac{\pi}{3} - \theta\right)\right\} = \theta + \left(\frac{\pi}{3} - \theta\right) = \frac{\pi}{3} = \text{R.H.S.}$$

29. We have,

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{\pi}{2}\right)\right]$$

Let $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = x \Rightarrow \tan x = \frac{-1}{\sqrt{3}} = -\tan\frac{\pi}{6}$

$$\Rightarrow \ \tan x = \tan\left(\pi - \frac{\pi}{6}\right) = \tan\frac{5\pi}{6} \ \Rightarrow \ x = \frac{5\pi}{6}$$

Also, $\cot^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{3}$

Similarly, $\tan^{-1}\left[\sin\left(\frac{\pi}{2}\right)\right] = \tan^{-1}(1) = \pi/4$

$$\therefore \ \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\frac{\pi}{2}\right)$$

$$= \frac{5\pi}{6} + \frac{\pi}{3} + \frac{\pi}{4} = \frac{17\pi}{12}$$

30. Here, $A=\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $B=[-1,1]$

Also $f: A \to B$ such that $f(x)=\sin x$

$\therefore$ f is one-one.

$\because$ $f(x_1)=f(x_2) \Rightarrow \sin x_1=\sin x_2$

$\Rightarrow$ $x_1=x_2$ $\left[\because x_1,x_2 \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right]$

Also, range $(f)=[-1, 1]=B$ so, f is onto.

Thus, f is one-one and onto and hence bijective.

31. We have, $A=\{x \in Z: 0 \le x \le 12\}$

$\therefore$ $A=\{0, 1, 2, 3, \ldots, 12\}$

and $S=\{(a, b): |a-b| \text{ is a multiple of } 4\}$

(i) Reflexive : For any $a \in A$,

$|a-a|=0$, which is a multiple of 4.

Thus, $(a, a) \in S$

$\therefore$ S is reflexive.

(ii) Symmetric : For any $a, b \in A$,

Let $(a, b) \in S$

$\Rightarrow$ $|a-b|$ is a multiple of 4.

$\Rightarrow$ $|b-a|$ is a multiple of 4 $\Rightarrow$ $(b, a) \in S$

i.e., $(a, b) \in S \Rightarrow (b, a) \in S$

$\therefore$ S is symmetric.

(iii) Transitive : For any $a, b, c \in A$,

Let $(a, b) \in S$ and $(b, c) \in S$

$\Rightarrow$ $|a-b|$ is a multiple of 4 and $|b-c|$ is a multiple of 4.

$\Rightarrow$ $a-b=\pm 4k_1$ and $b-c=\pm 4k_2$; $k_1, k_2 \in N$

$\Rightarrow$ $(a-b)+(b-c)=\pm 4(k_1+k_2)$; $k_1, k_2 \in N$

$\Rightarrow$ $a-c=\pm 4(k_1+k_2)$; $k_1, k_2 \in N$

$\Rightarrow$ $|a-c|$ is a multiple of $4 \Rightarrow (a, c) \in S$

$\therefore$ S is transitive.

Hence, S is an equivalence relation.

The set of elements related to 1 is $\{5, 9\}$.

OR

Consider any $(a, b), (c, d), (e, f) \in N \times N$.

(i) Since $a+b=b+a$ for all $a, b \in N$

(Commutative law of addition in N)

$\Rightarrow$ $(a, b)\, R\, (a, b) \Rightarrow R$ is reflexive.

(ii) Let $(a, b)\, R\, (c, d) \Rightarrow a+d=b+c \Rightarrow d+a=c+b$

$\Rightarrow$ $c+b=d+a \Rightarrow (c, d)\, R\, (a, b)$

Thus, $(a, b)\, R\, (c, d) \Rightarrow (c, d)\, R\, (a, b) \Rightarrow R$ is symmetric.

(iii) Let $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$

$\Rightarrow$ $a+d=b+c$ and $c+f=d+e$

$\Rightarrow$ $a+d+c+f=b+c+d+e$

$\Rightarrow$ $a+f=b+e \Rightarrow (a, b)\, R\, (e, f)$.

Thus, $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f) \Rightarrow (a, b)\, R\, (e, f)$

$\Rightarrow$ R is transitive.

Therefore, the relation R is reflexive, symmetric and transitive, and hence it is an equivalence relation.

32. Given $A=\mathbb{R}-\{3\}$; $B=\mathbb{R}-\{1\}$

and $f: A \to B$ defined as

$$f(x)=\left(\frac{x-2}{x-3}\right) \forall\, x \in A.$$

Here, f is defined $\forall\, x \in A$, as $3 \notin A$.

Also, $f(x) \ne 1$

$\left[\because f(x)=1 \Leftrightarrow \frac{x-2}{x-3}=1 \Leftrightarrow x-2=x-3 \Leftrightarrow -2=-3, \text{ which is absurd}\right]$

Let $x_1, x_2 \in A$ be such that $f(x_1)=f(x_2)$

$\Rightarrow$ $\frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3}$

$\Rightarrow$ $(x_1-2)(x_2-3)=(x_1-3)(x_2-2)$

$\Rightarrow$ $x_1x_2-3x_1-2x_2+6=x_1x_2-2x_1-3x_2+6$

$\Rightarrow$ $-3x_1-2x_2=-2x_1-3x_2 \Rightarrow x_1=x_2$

$\therefore$ f is a one-one function.

Let $y \in B=\mathbb{R}-\{1\} \Rightarrow y \ne 1$.

We want to solve $y=f(x)$ for some $x \in A$

$\Leftrightarrow$ $y=\frac{x-2}{x-3}$ $\Leftrightarrow$ $xy-3y=x-2$

$\Leftrightarrow$ $x(y-1)=-2+3y \Leftrightarrow$ $x=\frac{3y-2}{y-1} \in A$ (as $y \ne 1$)

$\therefore$ f is also onto.

$\Rightarrow$ $f: A \to B$ is a bijective function.

33. $$\sqrt{\frac{2-\sqrt{3}}{4}}=\sqrt{\frac{4-2\sqrt{3}}{4\times 2}}=\frac{\sqrt{(\sqrt{3}-1)^2}}{2\sqrt{2}}=\frac{\sqrt{3}-1}{2\sqrt{2}}=\sin\frac{\pi}{12}$$

$$\therefore\ \sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}=\sin^{-1}\left(\sin\frac{\pi}{12}\right)=\frac{\pi}{12}$$

$$\text{Also, } \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\cos^{-1}\left(\cos\frac{\pi}{6}\right)=\frac{\pi}{6}$$

$$\text{and } \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=\cos^{-1}\left(\cos\frac{\pi}{4}\right)=\frac{\pi}{4}$$

$$\therefore\ \cot^{-1}\left(\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)\right)$$

$$=\cot^{-1}\left(\cot\left(\frac{\pi}{12}+\frac{\pi}{6}+\frac{\pi}{4}\right)\right)=\cot^{-1}\left(\cot\left(\frac{\pi}{2}\right)\right)=\frac{\pi}{2}.$$

34. We have,

$$\tan^{-1}\left(\frac{1}{1+1\cdot 2}\right)+\tan^{-1}\left(\frac{1}{1+2\cdot 3}\right)+\ldots$$
$$+\tan^{-1}\left(\frac{1}{1+n\cdot(n+1)}\right)=\tan^{-1}\theta$$

$$\Rightarrow \tan^{-1}\left(\frac{2-1}{1+1\cdot 2}\right)+\tan^{-1}\left(\frac{3-2}{1+2\cdot 3}\right)+\ldots$$
$$+\tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)=\tan^{-1}\theta$$

$$\Rightarrow \quad \tan^{-1}2-\tan^{-1}1 + \tan^{-1}3-\tan^{-1}2+\ldots$$
$$+\tan^{-1}(n+1)-\tan^{-1}(n)=\tan^{-1}\theta$$

$$\Rightarrow \quad \tan^{-1}(n+1)-\tan^{-1}1=\tan^{-1}\theta$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{(n+1)-1}{1+\ (n+1)(1)}\right)=\tan^{-1}\theta$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{n}{n+2}\right)=\tan^{-1}\theta \ \Rightarrow \ \frac{n}{n+2}=\theta$$

OR

Consider,

$$\text{L.H.S.} = \tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}$$

Let $\cos^{-1}\frac{a}{b}=\theta \Rightarrow \frac{a}{b}=\cos\theta$

$$\therefore \quad \text{L.H.S.} = \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

$$=\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}+\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$$

$$=\frac{1+\tan^2\frac{\theta}{2}+1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}=2\left(\frac{1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right)$$

$$=\frac{2}{\cos\left(2\cdot\frac{\theta}{2}\right)}=\frac{2}{\cos\theta}=\frac{2}{a/b}=\frac{2b}{a}=\text{R.H.S.}$$

35. We have, $R = \{(a, b) : a \le b^2\}$, where $a, b \in \mathbb{R}$.

Reflexive : Since $\frac{1}{2}\le\left(\frac{1}{2}\right)^2$ is not true. Therefore, $\left(\frac{1}{2},\frac{1}{2}\right)\notin R$.

So, R is not reflexive.

Symmetric : Since, $-1 \le 3^2$ but $3 \not\le (-1)^2$

i.e., $(-1, 3) \in R$ but $(3, -1) \notin R$.

So, R is not symmetric.

Transitive : Since $2 \le (-3)^2$ and $-3 \le 1^2$ but $2 \not\le 1^2$

i.e., $(2, -3) \in R$ and $(-3, 1) \in R$ but $(2, 1) \notin R$.

So, R is not transitive.

36. Given A = Set of all polygons

$R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides$\}$

(i) Reflexive : Let $P \in A$

Clearly, number of sides of P = number of sides of P

$\therefore \quad (P, P) \in R, \forall P \in A$

Hence, R is reflexive.

(ii) Symmetric : Let $P_1, P_2 \in A$

Let $(P_1, P_2) \in R \Rightarrow P_1$ and P_2 have same number of sides

$\Rightarrow P_2$ and P_1 have same number of sides

$\Rightarrow (P_2, P_1) \in A$

Hence, R is symmetric.

(iii) Transitive : Let $P_1, P_2, P_3 \in A$

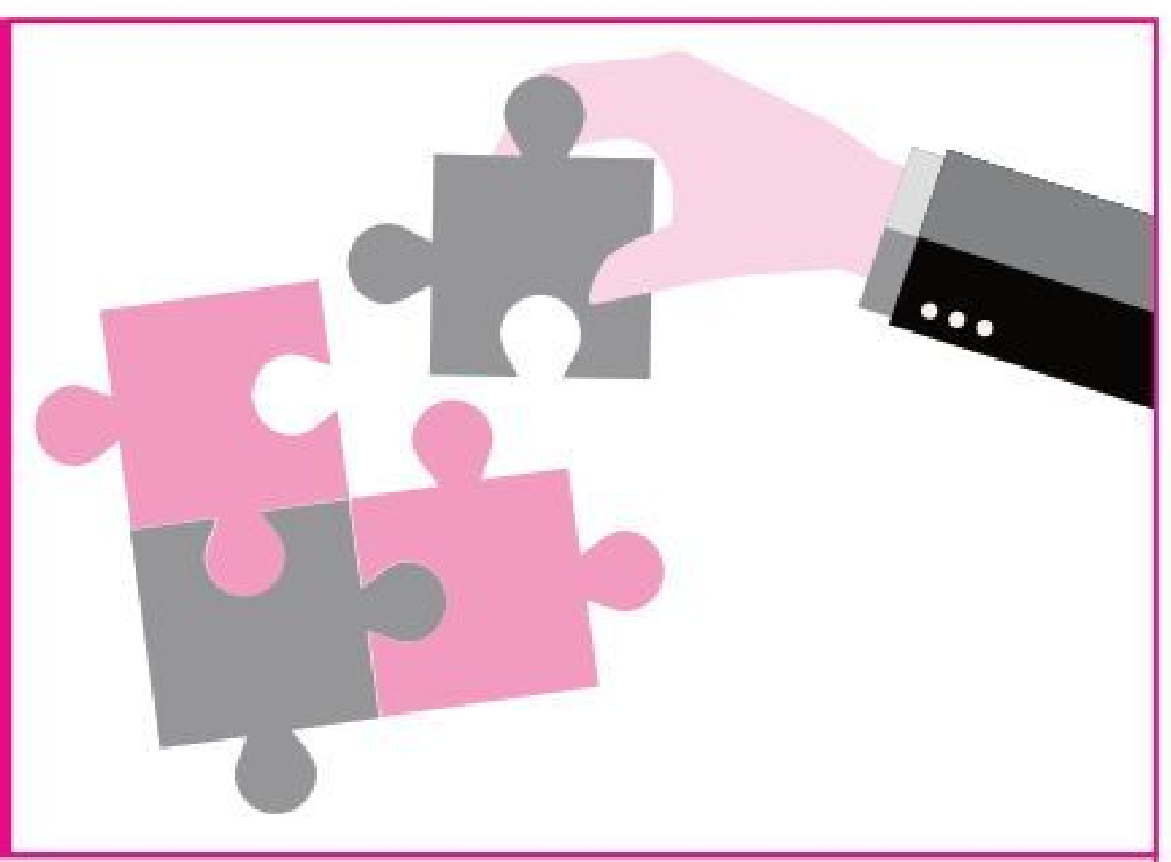
Let $(P_1, P_2) \in R$ and $(P_2, P_3) \in R$

$$\Rightarrow\left\{\begin{array}{l}\text{Number of sides of } P_1 = \text{number of sides of } P_2\\ \text{and number of sides of } P_2 = \text{number of sides of } P_3\end{array}\right\}$$

$\Rightarrow \quad$ Number of sides of P_1 = number of sides of P_3

$\Rightarrow \quad (P_1, P_3) \in R$

PUZZLE CORNER

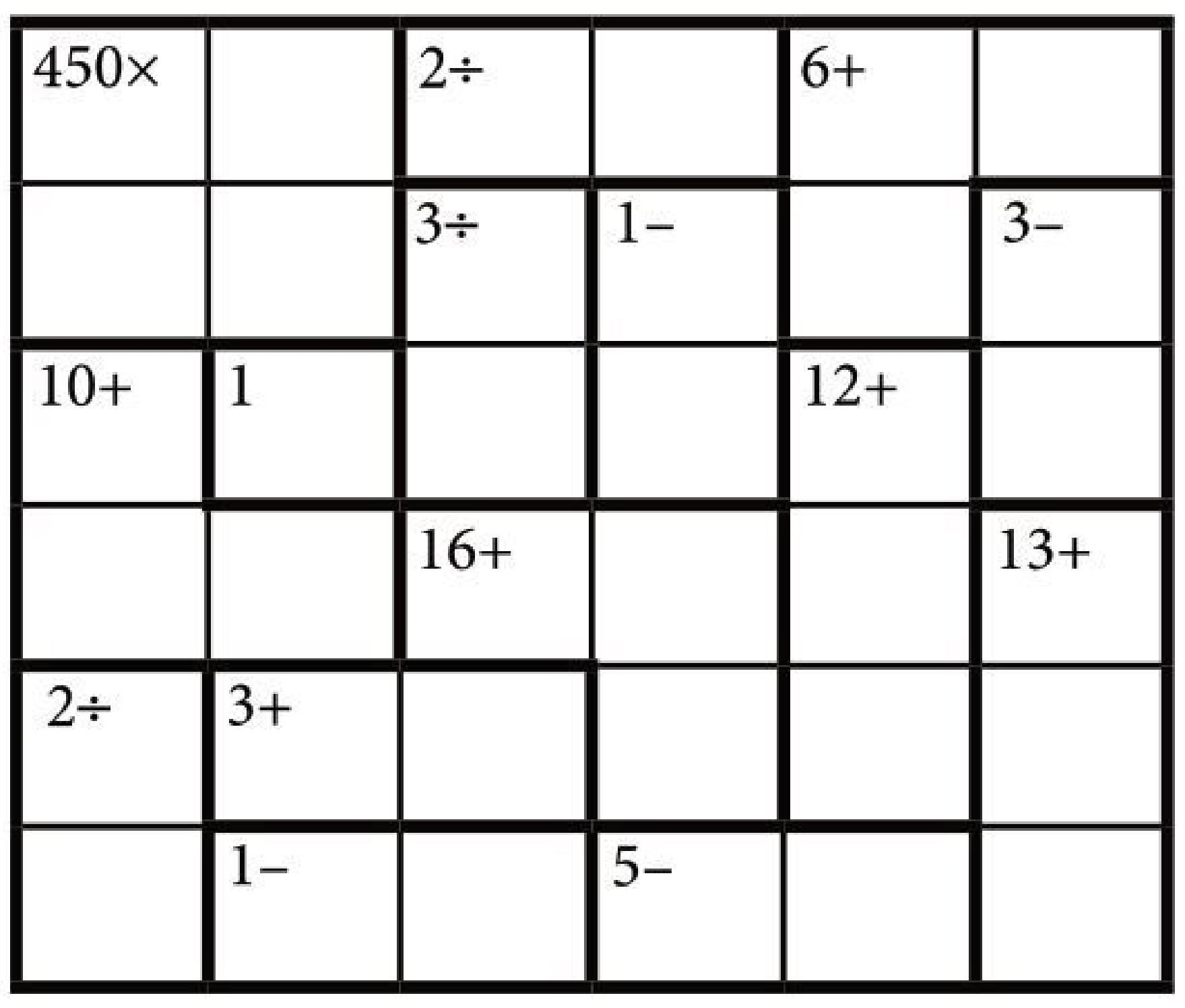


MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is multiple of its numbers. For example, if that value is 3 for a two-box cluster, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



450×		2÷		6+	
		3÷	1–		3–
10+	1			12+	
		16+			13+
2÷	3+				
	1–		5–		

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Hence, R is transitive.

Thus R is reflexive, symmetric and transitive and hence R is an equivalence relation on A.

Given T = a right angle triangle with sides 3, 4, 5

$\therefore$ Number of sides of $T = 3$

Hence all polygons having 3 sides will be R-related to T

$\therefore$ All triangles will be R-related to T

Hence, set of all elements in A which are R-related to T = Set of all triangles.

OR

(i) Reflexivity : Let (a, b) be an arbitrary element of $N \times N$. Then, $(a, b) \in N \times N$

$\Rightarrow \quad ab(b + a) = ba(a + b)$

[by commutativity of addition and multiplication on N]

$\Rightarrow \quad (a, b)\, R\, (a, b)$

So, R is reflexive on $N \times N$.

(ii) Symmetry : Let $(a, b), (c, d) \in N \times N$ be such that $(a, b)\, R\, (c, d)$.

$\Rightarrow \quad ad(b + c) = bc(a + d) \Rightarrow cb(d + a) = da(c + b)$

[by commutativity of addition and multiplication on N]

Thus, $(a, b)\, R\, (c, d) \Rightarrow (c, d)\, R\, (a, b)$ for all $(a, b), (c, d) \in N \times N$.

So, R is symmetric on $N \times N$.

(iii) Transitivity : Let $(a, b), (c, d), (e, f) \in N \times N$ be such that

$(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$. Then,

$(a, b)\, R\, (c, d) \Rightarrow ad(b + c) = bc(a + d)$

$$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad} \Rightarrow \frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d} \qquad \text{...(i)}$$

and $(c, d)\, R\, (e, f) \Rightarrow cf(d + e) = de(c + f)$

$$\Rightarrow \frac{d+e}{de} = \frac{c+f}{cf} \Rightarrow \frac{1}{d} + \frac{1}{e} = \frac{1}{c} + \frac{1}{f} \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$$\left(\frac{1}{b} + \frac{1}{c}\right) + \left(\frac{1}{d} + \frac{1}{e}\right) = \left(\frac{1}{a} + \frac{1}{d}\right) + \left(\frac{1}{c} + \frac{1}{f}\right)$$

$$\Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f} \Rightarrow \frac{b+e}{be} = \frac{a+f}{af}$$

$\Rightarrow \quad af(b + e) = be(a + f)$

$\Rightarrow \quad (a, b)\, R\, (e, f)$

So, R is transitive on $N \times N$.

Hence, R is an equivalence relation.

37. We have, $f(n) = n - (-1)^n$ for all $n \in N$.

$$\Rightarrow \quad f(n) = \begin{cases} n-1, \text{ if } n \text{ is even} \\ n+1, \text{if } n \text{ is odd} \end{cases}$$

One-one : Let n, m be any two even natural numbers, then $f(n) = f(m) \Rightarrow n - 1 = m - 1 \Rightarrow n = m$

If n, m are any two odd natural numbers, then $f(n) = f(m) \Rightarrow n + 1 = m + 1 \Rightarrow n = m$.

Thus in both the cases, $f(n) = f(m) \Rightarrow n = m$.

If n is even and m is odd, then $n \neq m$. Also $f(n)$ is odd and $f(m)$ is even. So, $f(n) \neq f(m)$.

Thus, $n \neq m \Rightarrow f(n) \neq f(m)$.

So, f is one-one function.

Onto : Let n be an arbitrary natural number.

If n is an odd natural number, then there exists an even natural number $n + 1$ such that

$f(n + 1) = n + 1 - 1 = n$

If n is an even natural number, then there exists an odd natural number $(n - 1)$ such that

$f(n - 1) = n - 1 + 1 = n$

Thus, every n $(\in N)$ has its pre-image in N.

So, $f : N \to N$ is onto.

Hence, $f : N \to N$ is a bijection.

OR

Given, $f(x) = \dfrac{x}{1+|x|}, x \in (-\infty, 0)$

$= \dfrac{x}{1-x} \qquad (\because x \in (-\infty, 0), |x| = -x)$

For one-one : Let $f(x_1) = f(x_2)$, $x_1, x_2 \in (-\infty, 0)$

$$\Rightarrow \frac{x_1}{1-x_1} = \frac{x_2}{1-x_2} \Rightarrow x_1(1 - x_2) = x_2(1 - x_1)$$

$\Rightarrow \quad x_1 - x_1x_2 = x_2 - x_1x_2 \Rightarrow x_1 = x_2$

Hence, if $f(x_1) = f(x_2)$, then $x_1 = x_2$

$\therefore$ f is one-one

For onto : Let $f(x) = y$

$$\Rightarrow y = \frac{x}{1-x} \Rightarrow y(1 - x) = x \Rightarrow y - xy = x$$

$$\Rightarrow x + xy = y \Rightarrow x(1 + y) = y \Rightarrow x = \frac{y}{1+y}$$

Here, $y \in (-1, 0)$

So, x is defined for all values of y.

$\therefore$ f is onto

38. We have,

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) \qquad \text{...(1)}$$

$$\text{Let } \sin^{-1}\left(\frac{2a}{1+a^2}\right) = p \Rightarrow \sin p = \frac{2a}{1+a^2} \qquad \text{...(2)}$$

Put $a = \tan\theta$ in (2), we get

$$\sin p = \frac{2\tan\theta}{1+\tan^2\theta} \Rightarrow \sin p = \sin 2\theta$$

$\Rightarrow \quad p = 2\theta \Rightarrow p = 2\tan^{-1} a$

$$\text{So, } \sin^{-1}\left(\frac{2a}{1+a^2}\right) = 2\tan^{-1} a \qquad \text{...(3)}$$

Similarly, $\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)=q \Rightarrow \frac{1-a^2}{1+a^2}=\cos q$ ...(4)

$\Rightarrow \cos q=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$ (Put $a = \tan\theta$ in (4))

$\Rightarrow q = 2\theta$

$\Rightarrow \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)=2\tan^{-1} a$ ...(5)

Now, let $\tan^{-1}\left(\frac{2x}{1-x^2}\right)=m \Rightarrow \tan m=\frac{2x}{1-x^2}$ ...(6)

Put $x = \tan\phi$ in (6), we get

$\tan m=\frac{2\tan\phi}{1-\tan^2\phi}=\tan 2\phi \Rightarrow m = 2\phi$

$\Rightarrow \tan^{-1}\left(\frac{2x}{1-x^2}\right)=2\tan^{-1} x$ ...(7)

From (3), (5) & (7), we get

$2\tan^{-1} a + 2\tan^{-1} a = 2\tan^{-1} x \Rightarrow 4\tan^{-1} a = 2\tan^{-1} x$

$\Rightarrow 2\tan^{-1} a = \tan^{-1} x$ ...(8)

Put $\tan^{-1} a = t$ in (8), we get

$2t = \tan^{-1} x \Rightarrow \tan(2t) = x$

$\Rightarrow x=\frac{2\tan t}{1-\tan^2 t} \Rightarrow x=\frac{2a}{1-a^2}$

OR

Consider, R.H.S. $= 2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$

Let $\tan^{-1} x = \theta \Rightarrow \tan\theta = x$ ...(i)

Now, $\cos 2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1-x^2}{1+x^2}$ (Using (i))

$\Rightarrow 2\tan^{-1} x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\therefore$ R.H.S. $=\cos^{-1}\left\{\frac{1-\tan^2(\alpha/2)\tan^2(\beta/2)}{1+\tan^2(\alpha/2)\tan^2(\beta/2)}\right\}$

$$=\cos^{-1}\left\{\frac{\cos^2(\alpha/2)\cos^2(\beta/2)-\sin^2(\alpha/2)\sin^2(\beta/2)}{\cos^2(\alpha/2)\cos^2(\beta/2)+\sin^2(\alpha/2)\sin^2(\beta/2)}\right\}$$

$$=\cos^{-1}\left\{\frac{(2\cos^2(\alpha/2))(2\cos^2(\beta/2))-(2\sin^2(\alpha/2))\cdot(2\sin^2(\beta/2))}{(2\cos^2(\alpha/2))(2\cos^2(\beta/2))+(2\sin^2(\alpha/2))\cdot(2\sin^2(\beta/2))}\right\}$$

$$=\cos^{-1}\left\{\frac{(1+\cos\alpha)(1+\cos\beta)-(1-\cos\alpha)(1-\cos\beta)}{(1+\cos\alpha)(1+\cos\beta)+(1-\cos\alpha)(1-\cos\beta)}\right\}$$

$$=\cos^{-1}\left(\frac{\cos\alpha+\cos\beta}{1+\cos\alpha\cos\beta}\right)=\text{L.H.S.}$$

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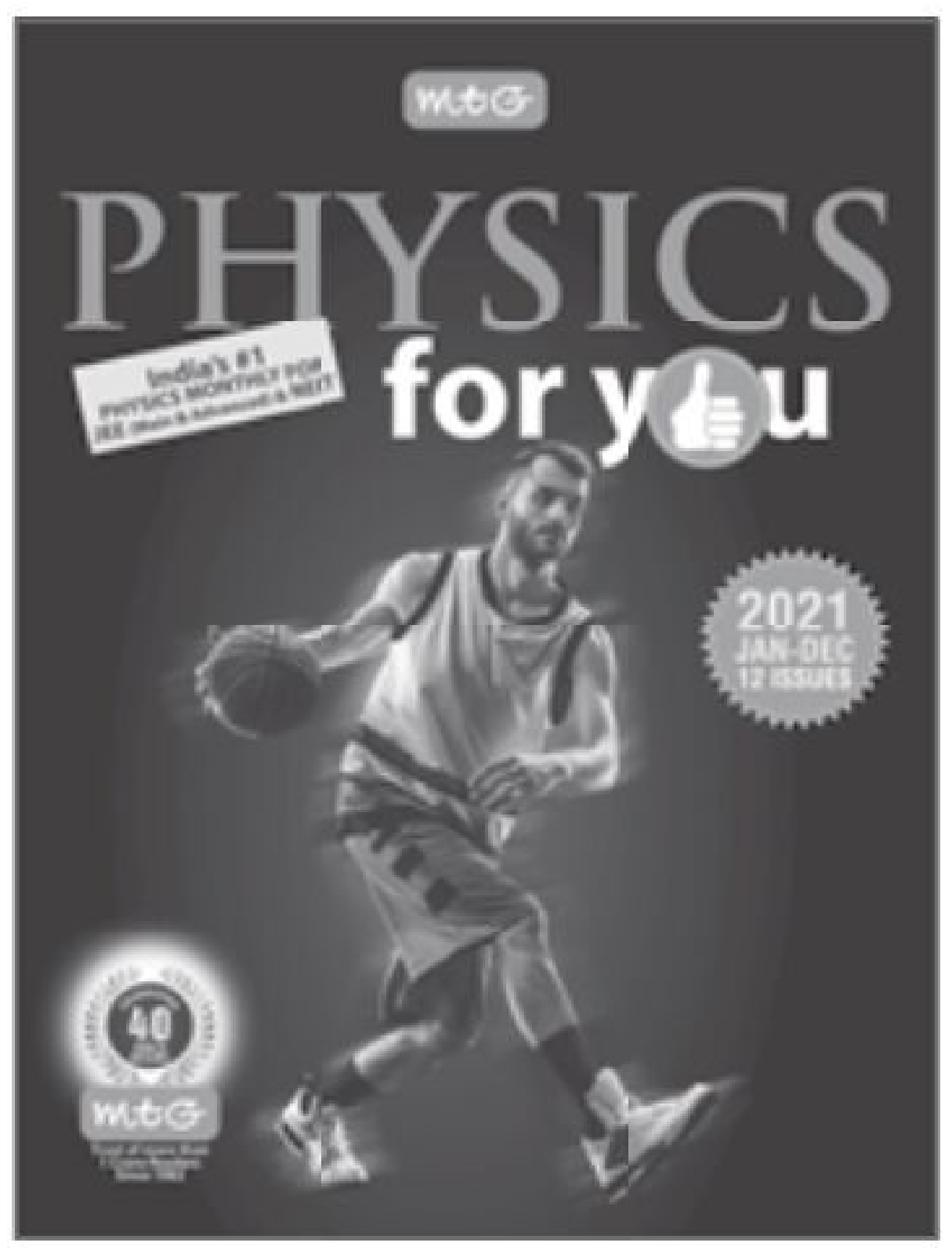
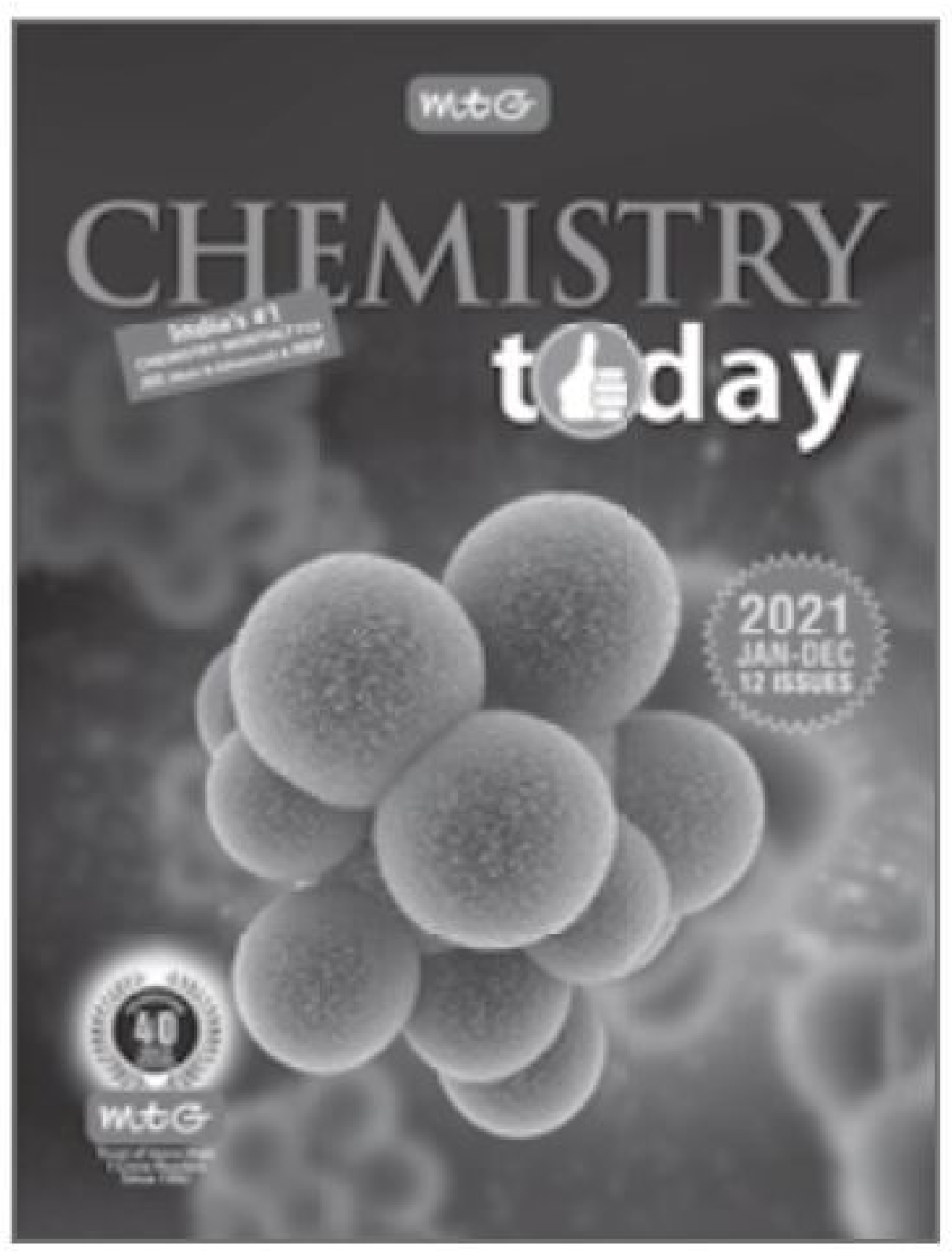
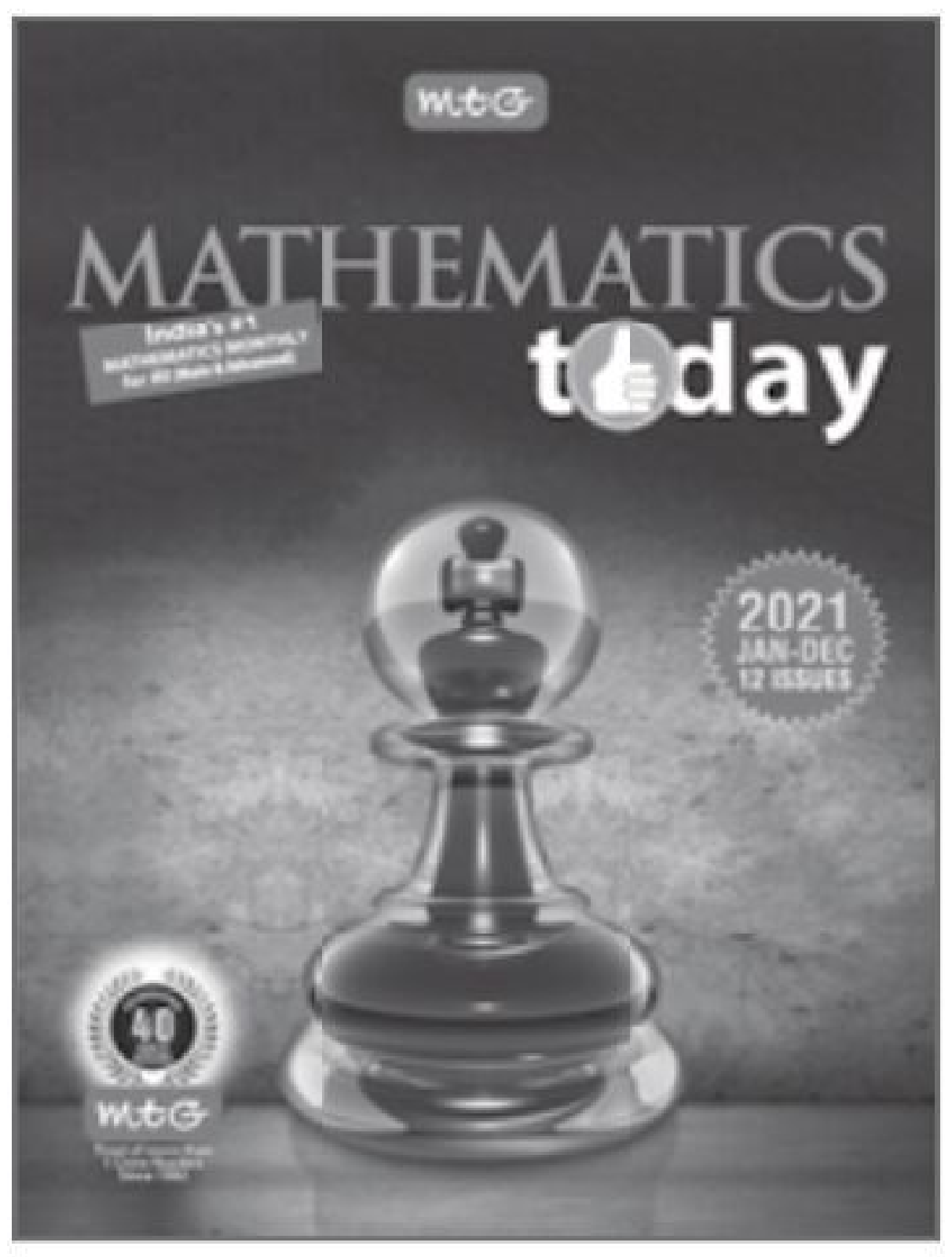
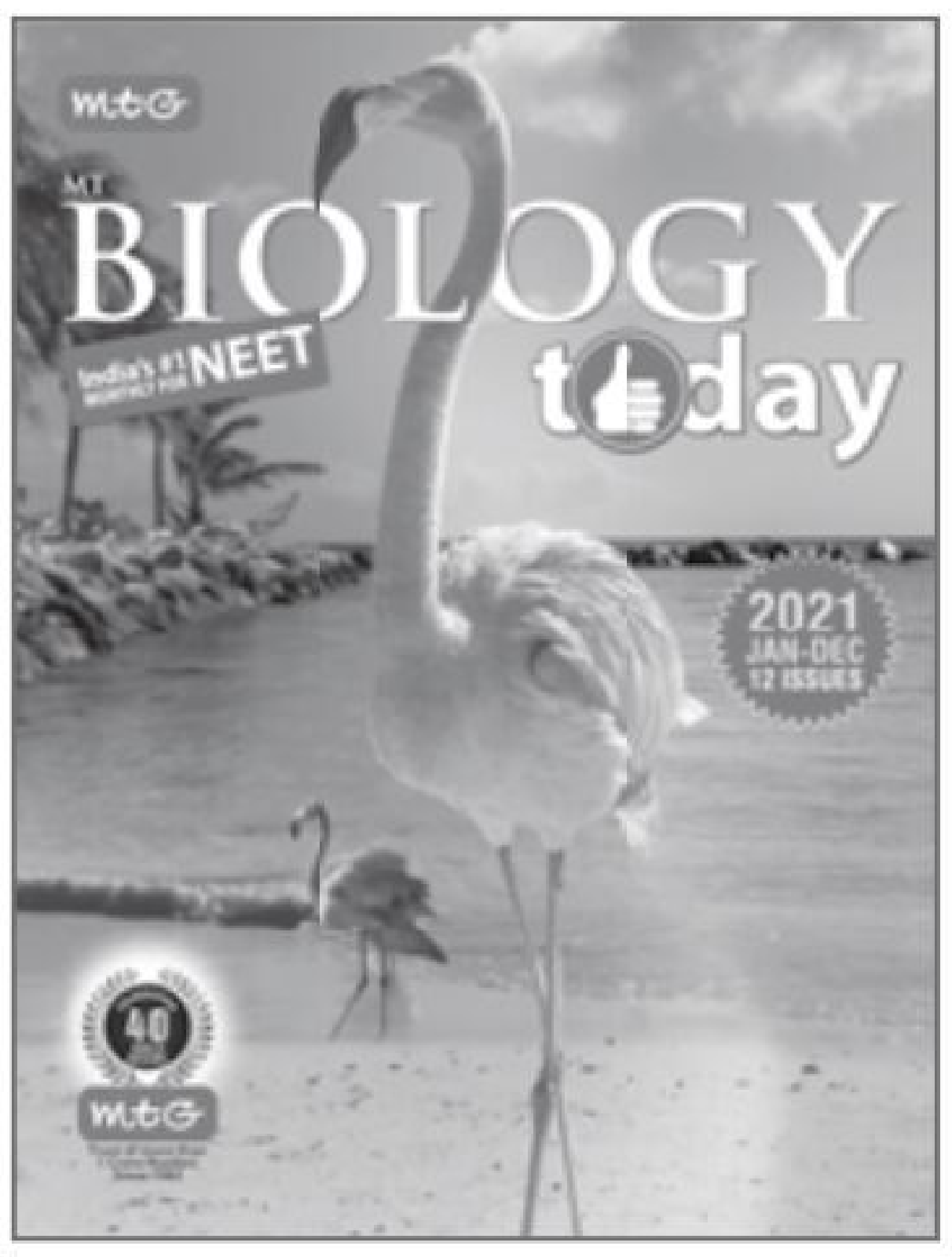

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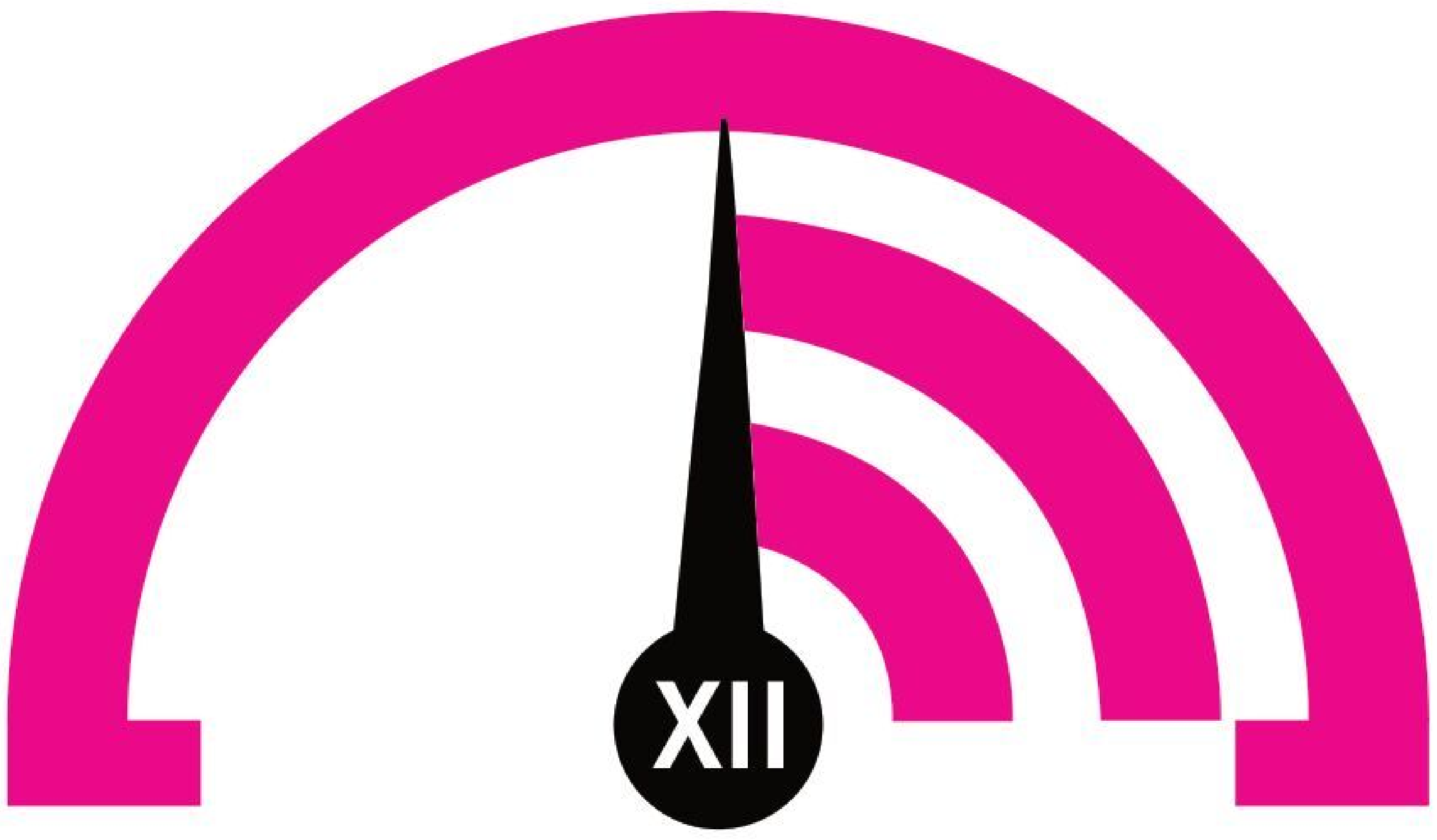
MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 1 : Relations & Functions

Time Taken : 60 Min.

Only One Option Correct Type

1. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is

(a) a function (b) transitive

(c) not symmetric (d) reflexive

2. $f : R \rightarrow R$ given by $f(x) = 2x + |\cos x|$, is

(a) one-one and into (b) one-one and onto

(c) many-one and into (d) many-one and onto

3. If $*$ is defined on the set R of all real numbers by $a * b = \sqrt{a^2 + b^2}$, then which of the following is true?

(a) 0 is the identity element in R w.r.t. $*$

(b) inverse of an element a in R w.r.t. $*$ is $1/b$, $b \neq 0$

(c) $*$ is not commutative

(d) none of these

4. Let R be the real line. Consider the following subsets of the plane $R \times R$:

$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$,

$T = \{(x, y) : x - y \text{ is an integer}\}$.

Which one of the following is true?

(a) S is an equivalence relation on R but T is not.

(b) T is an equivalence relation on R but S is not.

(c) Neither S nor T is an equivalence relation on R.

(d) Both S and T are equivalence relations on R.

5. Let $f(x) = \log_{x^2} 25$ and $g(x) = \log_x 5$. Then, $f(x) = g(x)$ holds for x belonging to

(a) R (b) $\{x : 0 < x < \infty, x \neq 1\}$

(c) ϕ (d) none of these

6. If $f : [2, \infty) \rightarrow [1, \infty)$ is defined by $f(x) = 2^{x^4 - 4x^2}$, then $f^{-1}(x)$ is

(a) $\sqrt{2 + \sqrt{4 - \log_2 x}}$ (b) $\sqrt{2 + \sqrt{4 + \log_2 x}}$

(c) $\sqrt{2 - \sqrt{4 \log_2 x}}$ (d) $\sqrt{2 + \sqrt{4 \log_2 x}}$

One or More Than One Option(s) Correct Type

7. Let $A = \{x \in R : x \geq 1/2\}$ and $B = \{x \in R : x \geq 3/4\}$. If $f : A \rightarrow B$ is defined as $f(x) = x^2 - x + 1$, then the solution set of the equation $f(x) = f^{-1}(x)$ is

(a) $\{1\}$ (b) $\{2\}$

(c) $\{1/2\}$ (d) none of these

8. Domain of $f(x) = \sin^{-1}[2 - 4x^2]$ is ($[\cdot]$ denotes the greatest integer function)

(a) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right] - \{0\}$ (b) $\left[-\frac{\sqrt{3}}{2}, 0\right)$

(c) $\left[-\frac{\sqrt{3}}{2}, 0\right) \cup \left(0, \frac{\sqrt{3}}{2}\right]$ (d) $\left[-\frac{\sqrt{3}}{2}, 8\right]$

9. Let f be the greatest integer function and g be the modulus function, then

(a) $(gof - fog)\left(-\frac{5}{3}\right) = 1$ (b) $(f + 2g)(-1) = 1$

(c) $(gof - fog)\left(\frac{5}{3}\right) = 0$ (d) $(f + 2g)(1) = 1$

10. Let $f(x) = 2x - \sin x$ and $g(x) = \sqrt[3]{x}$, then

(a) range of gof is R (b) gof is one-one

(c) both f and g are one-one

(d) both f and g are not one-one

11. Let $A = Q \times Q$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Then, which of the following is true?
(a) the identity element in A is $(1, 0)$
(b) the inverse of (a, b) is $\left(\frac{1}{a}, -\frac{b}{a}\right)$ if $a \neq 0$
(c) $*$ is not commutative
(d) none of these

12. If $f(x) = \sin^2 x + \sin^2\left(x+\frac{\pi}{3}\right) + \cos x \cdot \cos\left(x+\frac{\pi}{3}\right)$ and $g(5/4) = 1$, then $(gof)\,x$ is
(a) a polynomial of the first degree in $\sin x$, $\cos x$
(b) a constant function
(c) a polynomial of the second degree in $\sin x$, $\cos x$
(d) none of these

13. Let $f(x) = \sec^{-1}[1 + \cos^2 x]$, where $[\cdot]$ denotes the greatest integer function, then
(a) the domain of f is R
(b) the domain of f is $[1, 2]$
(c) the range of f is $[1, 2]$
(d) the range of f is $\{\sec^{-1} 1, \sec^{-1} 2\}$

Comprehension Type

Here $a_0 = x$, $a_{n+1} = f(a_n)$, where $n = 0, 1, 2, \ldots\ldots$.

14. If $f : R \to R$ be given by $f(x) = 3 + 4x$ and $a_n = A + Bx$, then which of the following is not true?
(a) $A + B + 1 = 2^{2n+1}$ (b) $|A - B| = 1$
(c) $\lim\limits_{n\to\infty} \frac{A}{B} = -1$ (d) none of these

15. If $f(x) = \frac{1}{1-x}$, then which of the following is not true?
(a) $a_n = \frac{1}{1-x}$, if $n = 3k + 1$
(b) $a_n = \frac{x-1}{x}$, if $n = 3k + 2$
(c) $a_n = x$, if $n = 3k$
(d) none of these

Matrix Match Type

16. Match the following :

	Column-I		Column-II
P.	$f : R \to \left[\frac{3\pi}{4}, \pi\right)$ and $f(x) = \cot^{-1}(2x - x^2 - 2)$, then $f(x)$ is	1.	one-one
Q.	$f : R \to R$ and $f(x) = e^{px} \sin qx$, where $p, q \in R^+$, then $f(x)$ is	2.	into
R.	$f : R^+ \to [4, \infty)$ and $f(x) = 4 + 3x^2$, then $f(x)$ is	3.	many-one
S.	$f : X \to X$ and $f(f(x)) = x\ \forall\ x \in X$, then $f(x)$ is	4.	onto

	P	Q	R	S
(a)	3, 4	3, 4	1, 2	1, 4
(b)	1, 4	1, 2	3, 4	2, 3
(c)	3, 4	1, 2	3, 4	1, 4
(d)	1, 4	3, 4	1, 2	3, 4

Numerical Answer Type

17. Let $n(A) = 4$ and $n(B) = 6$, then the number of one-one functions from A to B must be

18. Number of integers in the domain of function $f(x) + f(x^{-1}) = \frac{x^3+1}{x}$ is

19. If the number of different reflexive relations on a set A is equal to the number of different symmetric relations on set A, then the number of elements in set A is

20. Let $A = \{2, 3, 4, 5, \ldots, 17, 18\}$. Let '$R$' be the equivalence relation on $A \times A$, cartesian product of A with itself, defined by $(a, b)\ R\ (c, d)$ iff $ad = bc$. Then, the number of ordered pairs of the equivalence class of $(3, 2)$ is

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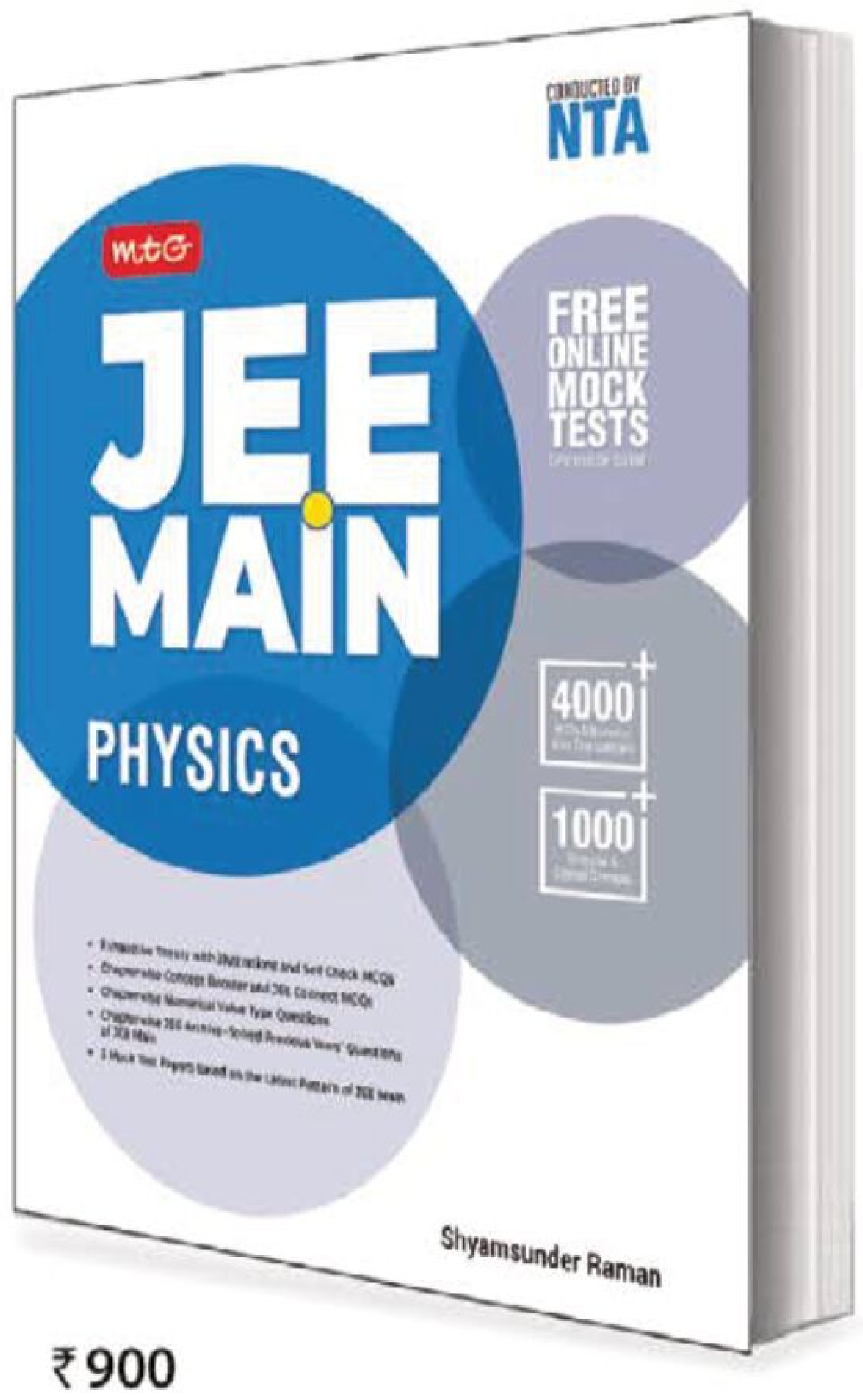
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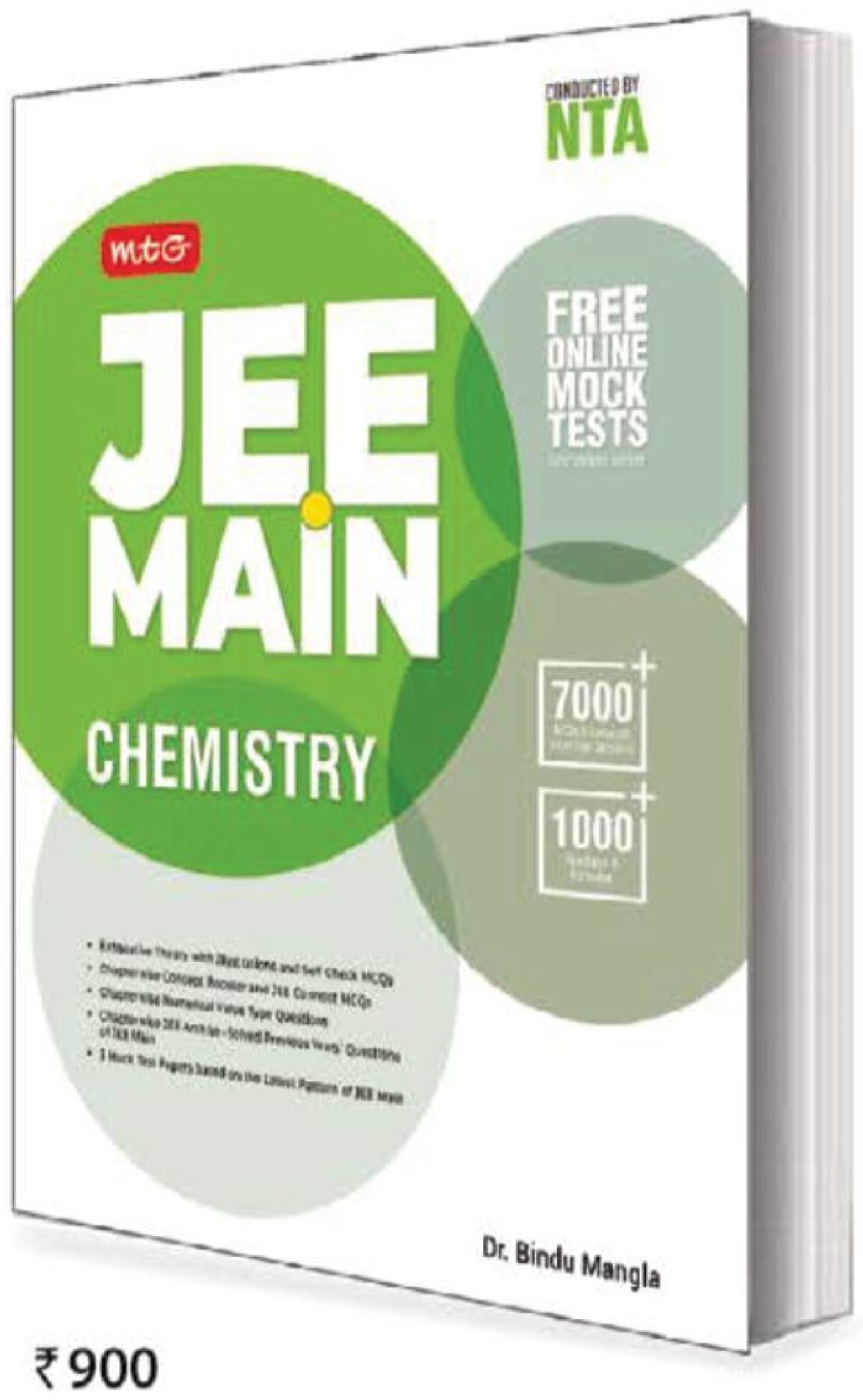
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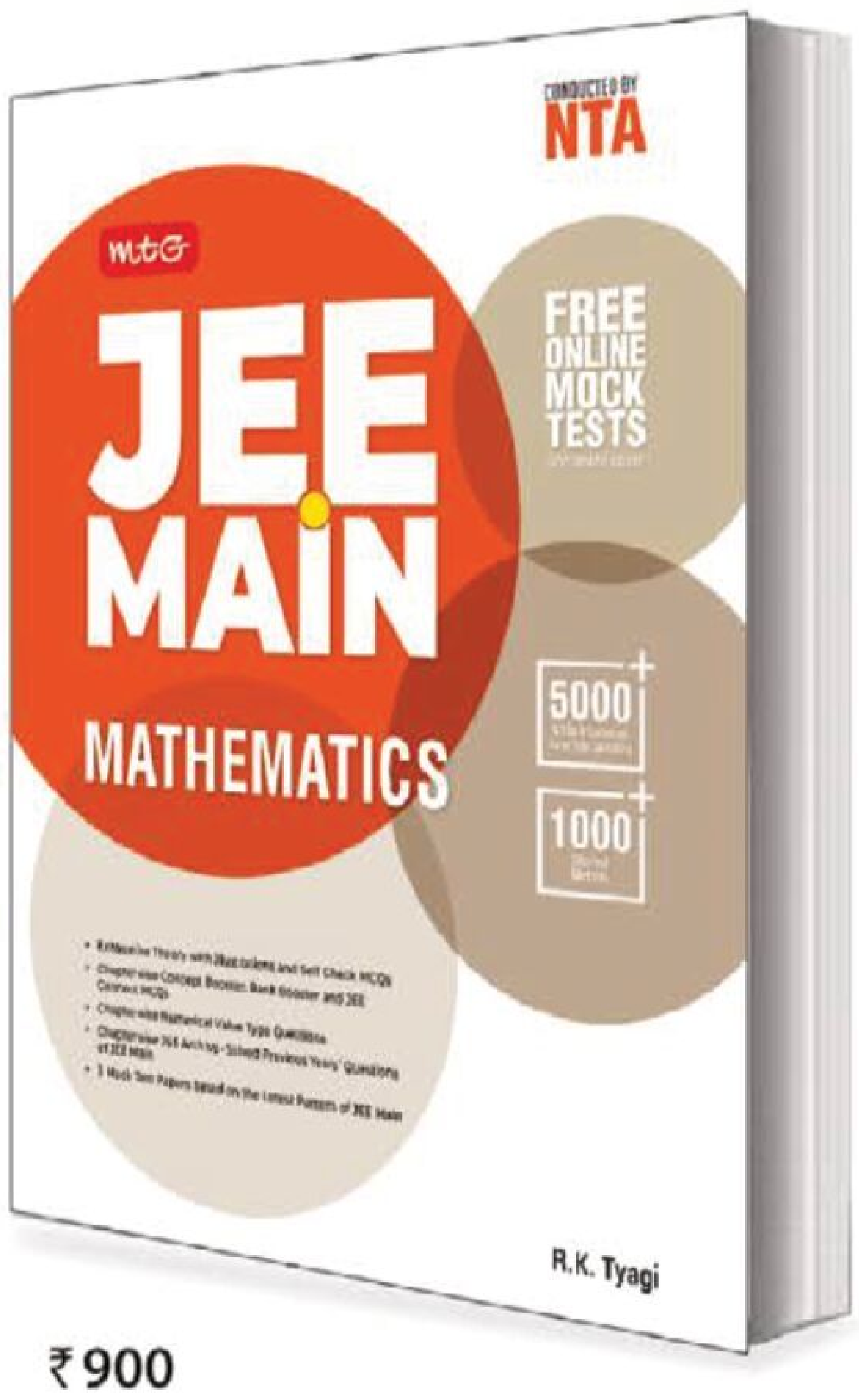
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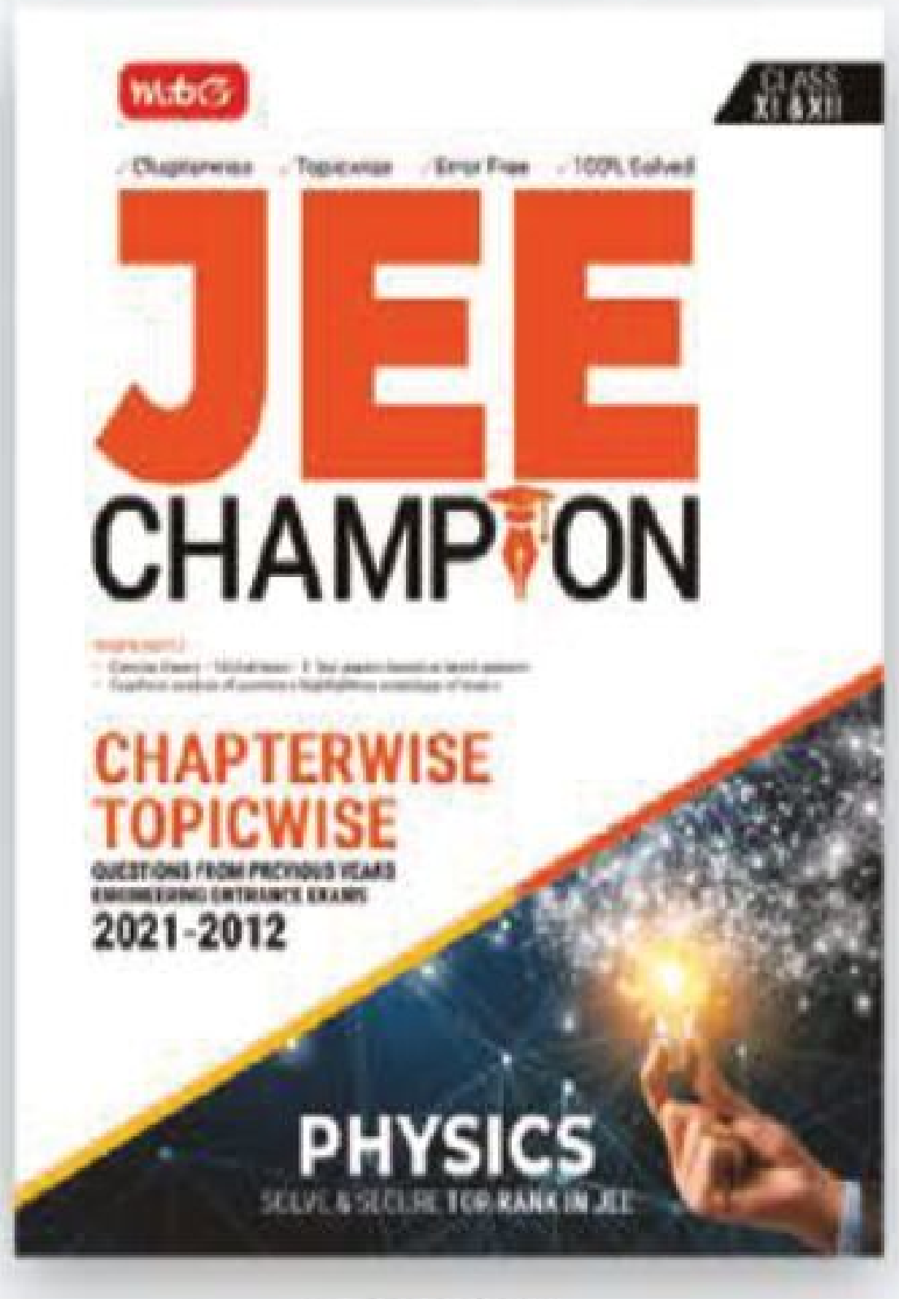
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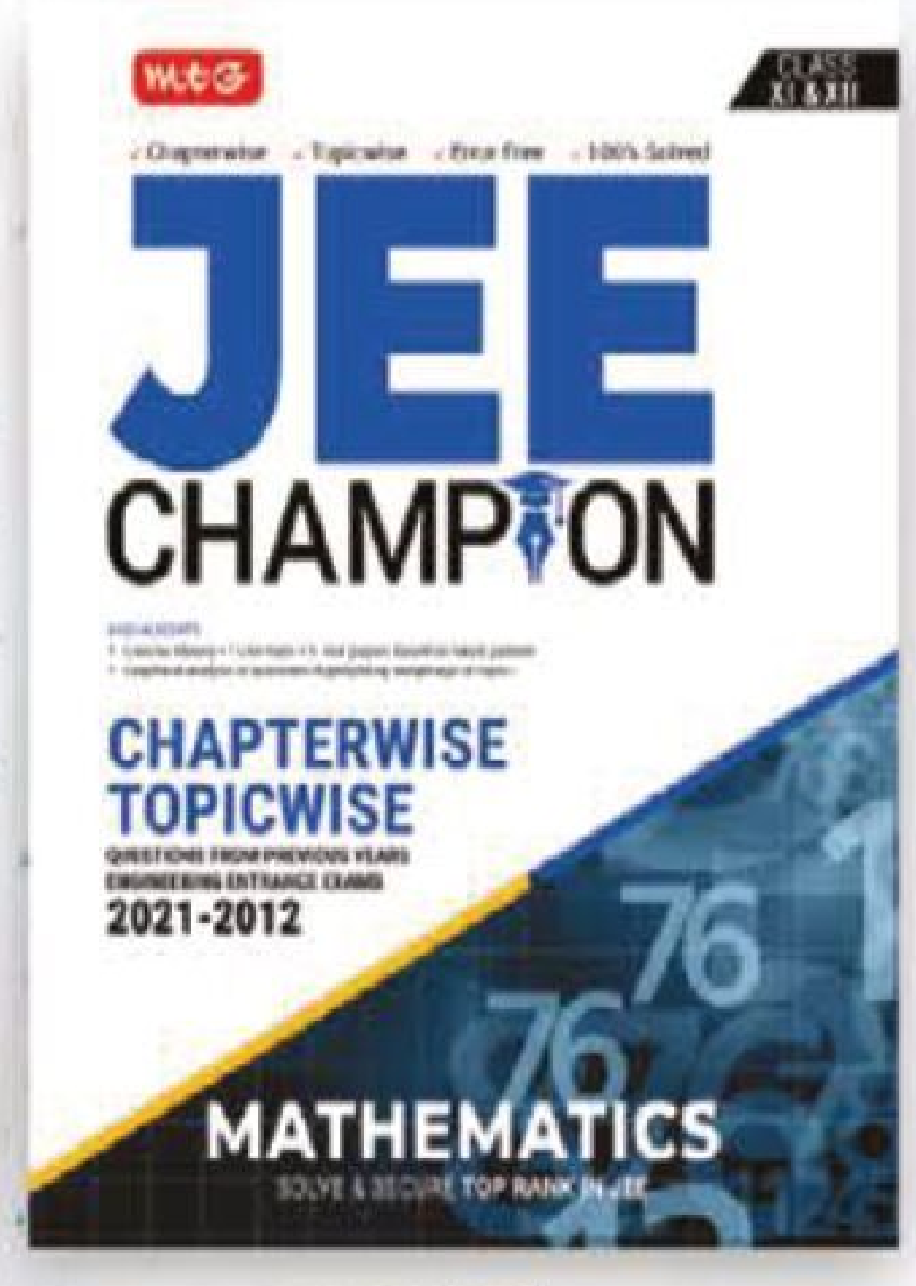
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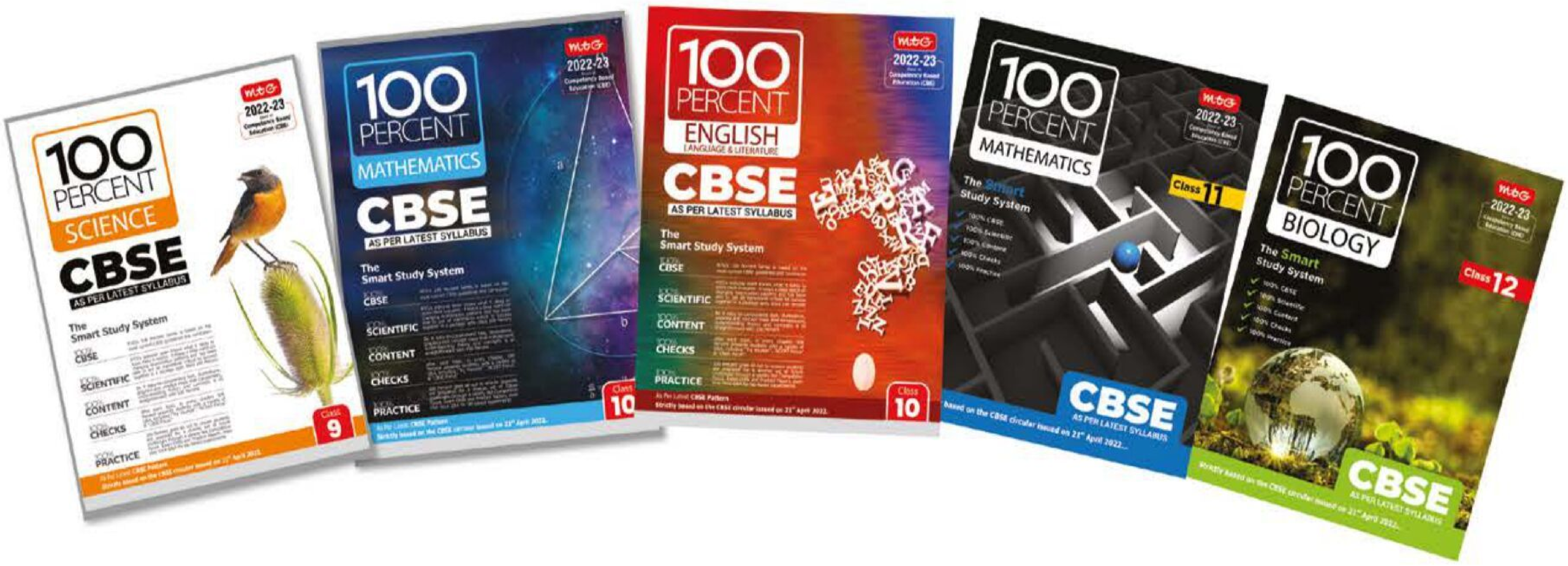
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